THE THERMOCAPILLARY CONVECTION IN LOCALLY HEATED LAMINAR LIQUID FILM FLOW CAUSED BY A CO-CURRENT GAS FLOW IN NARROW CHANNEL

Gatapova E.Y.*, gatapova@ngs.ru
Lyulin Y.V.*, yurl@ngs.ru
Marchuk I.V.*, marchuk@itp.nsc.ru
Kabov O.A.*, okabov@ulb.ac.be
Legros J.-C., jclegrros@ulb.ac.be

a) Institute of Thermophysics, Russian Academy of Sciences, prosp. Lavrentyev 1, Novosibirsk, 630090 Russia
b) Universite Libre de Bruxelles, Microgravity Research Center, Av. Roosevelt 50, B-1050 Brussels, Belgium
c) Euro Heat Pipes S.A., Rue de l’Industrie 24, 1400 - Nivelles, Belgium

ABSTRACT

A two-dimensional model of a steady laminar flow of liquid film and co-current gas flow in a plane channel is considered. It is supposed that the height of a channel is much less than its width. There is a local heat source on the bottom wall of the channel. An analytical solution for the temperature distribution problem in locally heated liquid film is obtained, when the velocity profile is linear. An analytical solution of the linearized equation for thermocapillary film surface deformation is found. A liquid bump caused by the thermocapillary effect in the region where thermal boundary layer reaches the film surface is obtained. Damped oscillations of the free surface may exist before the bump. This is obtained according to the solution of the problem in an inclined channel. It depends on the forces balance in the film. The defining criterion is found for this effect. The oscillations of free surface do not exist for horizontally located channel.

INTRODUCTION

The surface tension plays a crucial role in two-phase flow in mini- and microchannels (Kandlikar et al., 1999; Kawahara et al., 2002). The annular flow regime is one of the basic regimes of flow in mini- and microchannels also under microgravity. The laboratory investigation and theoretical modeling of such flow is possible using liquid film flow caused by a co-current gas flow (Kuznetsov, 2000; Kabov et al., 2001a). Temperature gradient on gas-liquid interface and concentration-capillary forces caused by concentration gradient of multicomponent liquids produce thermocapillary forces, which induce convection and intensive heat and mass transfer. The Marangoni effect may have an essential influence on heat transfer intensity and lead to film breakdown, dry spots formation, burn-out of heater. The aim of the work is to investigate of the influence of thermocapillary forces on hydrodynamics and heat transfer of liquid film flow in a plane channel 2-4 mm high. Applications of the investigations could be the cooling systems of microelectronic equipment, where heat is transferred from the integrated circuits to thin liquid film moving under friction of gas flow in a narrow gap (Sherwood and Cray, 1992). The heat transfer and hydrodynamics investigations in such system were performed by Bar-Cohen et al. (1995) and Bar-Cohen and Soldbreken (1996). The experiments were made under the flowing of Nitrogen and liquid FC-72 in a symmetrically heated 0.508 mm wide channel. For a superficial liquid velocity less than 1 cm/s the annular flow regime occurs at superficial gas velocity larger than 0.1 cm/s.

THE PROBLEM STATEMENT. BASIC EQUATIONS

We consider a channel with rectangular cross section, where the height $H$ is much less than its width $B$ ($H \ll B$). A layer of viscous incompressible liquid is moving in channel under the influence of tangential stress $\tau$, caused by the gas flow as well as under gravity force for a channel inclined by an angle $\varphi$ with respect to horizontal. A local heat source is situated on the bottom wall. The locality of heating means that the heat flux density is a finite function of variable $x$. A two-dimensional flow is considered. Let us choose the system of Cartesian coordinates ($x,y$) so that $Oy$ axis is orthogonal to the bottom wall of the channel and $Ox$ axis is directed along the gas flow. The coordinate origin is situated at the beginning of the heater. It is supposed that the surface tension depends on temperature

$$\sigma = \sigma_0 - \sigma_\tau (T - T_0).$$
The steady two-dimensional motion of liquid film is described by equations (1)-(4).

\[ \rho \left( u_x + v_n \right) = \mu \left( u_x + u_y \right) - p_x + \rho g \sin \varphi \]  

\[ \rho \left( u_y + v_n \right) = \mu \left( v_x + v_y \right) - p_y - \rho g \cos \varphi \]  

\[ u_x + u_y = 0 \]  

\[ u_T_x + v_T_y = a \left( T_{x} + T_{yy} \right) \]  

The boundary conditions are written in the following way:

\[ u(x,0) = v(x,0) = 0 \]  

is a non-slip condition.

\[ -\lambda T_y(x,0) = q(x) \]  

A local heat source is specified.

\[ h_x = \frac{v(x,h(x))}{u(x,h(x))} \]  

is a kinematic condition. The expression

\[ \left[ p_x + \sigma_{xx} \right] \vec{n} = \frac{\sigma}{R} \vec{n} - \sigma_{x} T \vec{T} + \tau \vec{T} \]  

describes the balance of forces acting on the surface of the liquid (Landau and Lifshits, 1986)

\[ -\lambda T_y(x,h(x)) = \alpha \left( T(x,h(x)) - T_s \right) \]  

is the condition of heat exchange on the film surface

\[ T(-\infty,y) = T_0 \]  

is the given initial temperature. Here

\[ \left( \sigma_{xx} \right) = -p \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \mu \begin{pmatrix} 2u_x & u_x + v_x \\ u_y + v_x & v_y \end{pmatrix} \]  

is the viscous stress tensor, \( R \) is the curvature radius of the free surface of the liquid, which is determined as

\[ \frac{1}{R} = \frac{h_y}{(1 + h_y)^{3/2}} \]

are the unit vectors of the tangent plane and of the outward normal to the surface of the liquid.

Making use of the scalar products of (8) by unit vectors of tangent plane and of outward normal to the surface, we obtain boundary conditions for the normal and tangential stresses on the free surface of the liquid:

\[ p - p_s = -\sigma \frac{h_x}{(1 + h_y)^{3/2}} + \frac{2\mu (v_y - u_y + v_y + h_y^2 u_x)}{(1 + h_y^2)} \]  

\[ \frac{\mu (2h_y (v_x - u_x) + 1 - h_y^2) (v_x + u_x)}{(1 + h_y^2)} + \sigma_{x} T_s - \tau = 0 \]  

\[ T(x,h(x)) = \left( \nabla T, \vec{T} \right) = \frac{T_s(x,h(x)) h_y}{\sqrt{1 + h_y^2}} + \frac{T_s(x)}{\sqrt{1 + h_y^2}} \]

where \( \vec{T}(x) = T(x,h(x)) \).

## Laminar Flow of Rigid Isothermal Film

It is easy to obtain a solution of the problem in case of non-deformable laminar film and gas flows. This solution gives a velocity profile, tangential stress and can be used for calculation of the liquid temperature. When the temperature of the film surface is calculated, it is possible to use it for obtaining the steady thermocapillary deformations because the variations of film thickness weakly influence the film surface temperature due to constant liquid flow rate in the cross section of the channel.

We consider an infinite channel with rectangular cross section, where the height of channel is \( h \) and the width is \( B \). A layer of viscous incompressible liquid and a viscous incompressible gas are moving in the channel. The average gas velocity is essentially higher than average liquid velocity. The motion of gas is caused by existence of constant pressure gradient along the longitudinal coordinate \( x \). A liquid is moving under the influence of pressure gradient and under tangential stress \( \tau \), caused by gas flow, as well as under gravity force for inclined at an angle \( \varphi \) to the horizon channel. It is assumed that \( B >> H \) and the flow is two-dimensional. The Navier-Stokes equations for steady laminar flow of viscous incompressible liquid are follows:

For the liquid film

\[ \mu \frac{d^2 u_x}{dy^2} + \rho g \sin \varphi \frac{dp}{dx} = 0 \]  

\[ -\rho g \cos \varphi = 0 \]  

for gas

\[ \mu \frac{d^2 u_y}{dy^2} - \frac{dp}{dx} = 0 \]
The average liquid velocity is

\[ u_l = \frac{1}{\mu_l} \left( \frac{dp}{dx} - \rho_l g \sin \phi \left( \frac{1}{2} y^2 - hy \right) + \tau y \right) \]  

(20)

a gas velocity

\[ u_g = \frac{1}{\mu_g} \left( \frac{dp}{dx} \left( y^2 - H^2 \right) - h(y-H) + \tau(y-H) \right) \]  

(21)

The average liquid velocity is

\[ U_l = \frac{Q_l}{h} \]  

(22)

the average gas velocity is

\[ U_g = \frac{Q_g}{H-h} \]  

(23)

Out of condition (18) and equations (20), (21) we have:

\[ \tau = \frac{1}{2} \frac{d}{dx} \left( \frac{\mu_l h^2}{\mu_l h - \mu_l (h-H)} \right) - \frac{1}{2} \frac{\mu_l h^2 \rho_l g \sin \phi}{\mu_l h - \mu_l (h-H)} \]  

(24)

Under the conditions of constancy of liquid and gas flow rates

\[ q_l = \iint_{0}^{h} u_l dz dy = \text{const}, \]  

(25)

\[ q_g = \iint_{0}^{h} u_g dz dy = \text{const} \]  

(26)

we obtain:

for liquid

\[ Q_l = \frac{1}{\mu_l} \left( \frac{h^3}{3} \frac{dp}{dx} - \rho_l g \sin \phi + \frac{\tau h^2}{2} \right), \]  

(27)

for gas

\[ Q_g = \frac{1}{\mu_g} \left( \frac{h^3}{3} \frac{dp}{dx} - \rho_g g \sin \phi + \frac{\tau h^2}{2} \right), \]  

(28)

where \( Q_l = \frac{q_l}{B} \), \( Q_g = \frac{q_g}{B} \) are flow rates of liquid and gas.

Reynolds numbers of liquid and gas are:

\[ \text{Re}(l) = \frac{Q_l \rho_l}{\mu_l} \]  

(29)

\[ \text{Re}(g) = \frac{Q_g \rho_g}{\mu_g} \]  

(30)

Assigning height \( H \), width \( B \) of channel; volumetric flow rates of liquid \( q_l \) and gas \( q_g \) and inclination angle \( \phi \) out of (24),(27),(28) one can obtain the drop of pressure \( dp/dx \), the tangential stress \( \tau \) and the film thickness \( h \).

Expressing \( dp/dx \) and \( \tau \) by way of \( h \) we obtain:

\[ \frac{dp}{dx} = \frac{12Q_l \mu_l \left( \mu_l h - \mu_l (h-H) \right) - 3\mu_l h^2 (h-H)^2 \rho_l g \sin \phi}{(h-H)^2 (\mu_l h - \mu_l (h-H) - 4\mu_l h H)}, \]  

(31)

\[ \tau = \frac{6Q_l \mu_l \left( \mu_l h - \mu_l (h-H) \right) - 2\mu_l h^2 (h-H)^2 \rho_l g \sin \phi}{(h-H)^2 (\mu_l h - \mu_l (h-H) - 4\mu_l h H)} \]  

(32)

The problem is solved numerically by bisection method. The value of \( h \) is varied. The initial approximation is taken \( h_0 = H/2 \). Consecutively out of equations (31),(32),(27) the appropriate \( dp/dx \), \( \tau \), \( Q_l \) are defined for current value \( h \). The property of monotonic dependence of flow rate \( Q_l \) from film thickness \( h \) is used for calculation next approximation. Substituting finally obtained values of the pressure drop, film thickness and tangential stress to the equations (20),(21) gives values of film velocity and gas velocity.

The calculations for regimes of the flow corresponding to the planning experiments have been executed. The flow of the FC-72 liquid film and Nitrogen in the channel 4x120 mm² is calculated for the range of flow rates \( q_l = 0.5-9.2 \times 10^{-5} \text{ m}^3/\text{s} \), \( q_g = 4 \times 10^{-4} \text{ m}^3/\text{s} \), \( \text{Re}(l)=0-1.13 \), \( \text{Re}(g)=0-242.8 \). Investigation is made for horizontally located channel and for inclined one at an angle of 5 degree to the horizon. For horizontally located channel the velocity profile in the film is practically linear, Fig. 2. The difference between tangential stresses on the wall and one on the gas-liquid interface is equal 8% (Fig. 3). It allows to make conclusion that the gas friction is the main moving force for the film. The influence of the pressure gradient along the channel on the liquid motion is not important. At \( q_l = 5.92 \times 10^{-5} \text{ m}^3/\text{s} \) and \( q_g = 4 \times 10^{-4} \text{ m}^3/\text{s} \) the liquid motion is caused mainly by gravity for inclined at an angle of 5 degree to the horizon channel position. The difference between tangential stresses on the wall and interface is amounted to 536% (Fig. 3). The velocity profile in this case is the parabolic one as shown in Fig. 2.

The film thickness dependence on an inclination angle of the channel for different Reynolds numbers of the liquid and constant gas flow rate is presented in Fig. 4. The behavior of curves shows that the influence of gravity on the film thickness is less for small \( \text{Re} \) numbers.
Fig. 2. The velocity profile in the film for different inclination angle of the channel. FC-72, Nitrogen, \( q_l = 5.05 \times 10^{-8} \text{ m}^3/\text{s}, \ q_g = 4 \times 10^{-4} \text{ m}^3/\text{s}, \ T_0 = 30^\circ \text{C}. \)

Fig. 3. The tangential stresses ratio versus the channel inclination angle to the horizon. FC-72, Nitrogen, \( q_l = 5.05 \times 10^{-8} \text{ m}^3/\text{s}, \ q_g = 4 \times 10^{-4} \text{ m}^3/\text{s}, \ T_0 = 30^\circ \text{C}. \)

Fig. 4. The film thickness versus the inclination angle for different Reynolds numbers of liquid. FC-72, Nitrogen, \( q_g = 4 \times 10^{-4} \text{ m}^3/\text{s}, \ T_0 = 30^\circ \text{C}. \)

THE TEMPERATURE DISTRIBUTION IN LOCALLY HEATED LIQUID FILM

We consider a heat transfer problem for rigid liquid film, i.e., \( h = h_0 = \text{const} \), when the velocity profile in the film is linear and \( \nu = 0 \). The existence of a local heat source with the constant heat flux density on the bottom wall of the channel is supposed

\[
q(x) = q_0(\chi(x) - \chi(x - L)),
\]

where \( q_0 = \text{const} \), \( L \) is the length of the heater and \( \chi(x) \) is Heaviside function. We assume that the thin layer approximation, i.e., \( \varepsilon = h_0 / L < 1 \), is valid. In the problem (4), (6), (9), (10) we change over to non-dimensional variables \( X, Y, U, \theta, \theta_e \) with the help of the formulas:

\[
X = \frac{x}{L}, \ Y = \frac{y}{h_0}, \ \theta = \frac{T - T_0}{\Delta T}, \ q_e = \frac{T_e - T_0}{\Delta T}, \ U = \frac{u}{\bar{u}},
\]

where \( \Delta T = \frac{q_L}{c_p \Gamma}, \ \bar{u} = \frac{1}{2 \mu} h_0, \ q(y) = 2\pi \frac{y}{h_0}. \)

Ignoring the terms of the order \( \varepsilon^2 \) and higher we rewrite the problem (4), (6), (9), (10) in the following form:

\[
\varepsilon Pe2Y \theta_x = \theta_{YY}, \quad \theta(0, Y) = 0 \quad (33)
\]

\[
-\frac{\lambda}{c_p \Gamma} \theta_\gamma(X,0) = \varepsilon (\chi(X) - \chi(X - 1)) \quad (35)
\]

\[
\theta_\gamma(X,1) = -Bi(\theta(X,1) - \theta_\varepsilon) \quad (36)
\]

Let us denote \( \eta(X) = \chi(X) - \chi(X - 1). \)

The solution of the problem (33)-(36) is (Mikhailov, 1972):

\[
\theta(X,Y) = \theta^0(X,Y) + \sum_{i=0}^{\infty} G_i \psi(Y) \exp(-\xi_i^2 X) \frac{1}{\xi_i^2} \left[ Bi \psi_i(0) - \frac{c_p \Gamma}{\lambda} \left( \int_0^X \exp(\xi_i^2 X) \frac{\partial \theta(X)}{\partial X} dX \right) \right] \quad (37)
\]

where

\[
\theta^0(X,Y) = \left[ 1 + \frac{1}{Bi} - Y \right] \frac{c_p \Gamma}{\lambda} \eta(X) + \theta_\varepsilon
\]

is a solution of the following problem:

\[
\theta^0_{YY}(X,Y) = 0 \quad (38)
\]

\[
-\frac{\lambda}{c_p \Gamma} \theta^0_\gamma(X,0) = \varepsilon \eta(X) \quad (39)
\]

\[
\theta^0_\gamma(X,0) = -Bi(\theta^0(X,1) - \theta_\varepsilon) \quad (40)
\]

\[
\psi_i(Y) = \sqrt{\frac{2}{\pi}} \frac{\sqrt{2 \varepsilon Pe \xi_i}}{\xi_i^2} Y^{\frac{3}{2}}, \quad \xi_i, \ i = 1, ..., \infty
\]

are eigenfunctions and eigenvalues of Sturm-Liouville’s problem:
falling down under the action of gravity theoretically and experimentally have been studied. The value Bi=0.00366 corresponds to the value of heat-transfer coefficient on the film surface $\alpha = 20 \text{ W/m}^2\text{K}$. There is a practically linear increase of temperature in the heater area. The influence of convective heat transfer mechanism is becoming stronger with increasing Re number. The length of the thermal boundary layer also increases with increasing Re number. The temperature reduction beyond the heater is explained by convective heat exchange between surface of the film and gas flow. The reduction of temperature is more noticeable at smaller Re numbers. The influence of heat exchange between surface of the film and gas flow on temperature of the liquid is shown in Fig. 6. A variation of Biot number in the range 0.005-1 corresponds to variation of heat-transfer coefficient on the film surface from 1.02 W/m²K to 203.1 W/m²K. The picture shows qualitatively a possible effect of evaporation of the liquid.

**THE SOLUTION OF LINEARIZED EQUATION FOR FILM THICKNESS**

The non-zero temperature gradient on the film surface leads to appearance of the thermocapillary effect. A tangential stress on the film surface deforms the film by reason of Marangoni effect. If the temperature distribution on the liquid surface is known, for instance out of analytical solution (37) or experiment, one can find a value of the deformations.

Let the tangential stress $\tau$ and initial film thickness $h_0$ be given. In the problem (1)-(10) we change over to dimensionless variables $X, Y, \theta, \theta_a, \bar{\theta}, U, V, P, T, \bar{h}$ with the help of the formulas:

$$
X = \frac{x}{L}, \quad Y = \frac{y}{h_0}, \quad \theta = \frac{T - T_0}{\Delta T}, \quad \theta_a = \frac{T_a - T_0}{\Delta T},
$$

$$
\bar{\theta} = \frac{T - T_0}{\Delta T}, \quad U = \frac{u}{\bar{u}}, \quad V = \frac{v}{\bar{u}}, \quad P = \frac{p - p_a}{p_0}, \quad T = \frac{\tau}{\tau_0},
$$

$$
\bar{h} = \frac{h}{h_0}, \quad \Delta T = \frac{q_y L}{c_y \Gamma}, \quad p_0 = \frac{\mu \bar{u}}{\varepsilon h_0}, \quad \tau_0 = \frac{\mu \bar{u}}{h_0},
$$

$$
\Sigma = \frac{\varepsilon \sigma \Delta T}{\mu \bar{u}}, \quad u(y) = 2\pi y\bar{u}, \quad \bar{u} = \bar{u}\bar{u}.
$$

For horizontally located channel the average velocity in film is expressed with the help of the tangential stress $\bar{u} = \frac{1}{2} \mu h_0$, and dimensionless tangential stress is $T = \frac{\tau}{\tau_0} = 2$.

Ignoring the terms of the order $\varepsilon^2$ and higher the system of equations (1)-(10) in the terms of dimensionless variables becomes:

$$
0 = \varepsilon U_{YY} - \varepsilon P_y + \frac{\rho gh_0}{p_0} \sin \varphi \tag{44}
$$

$$
0 = -P_y - \frac{\rho gh_0}{p_0} \cos \varphi \tag{45}
$$

The calculated temperature distribution on the film surface of the 10% solution of ethyl alcohol in water for different liquid flowrates is presented in Fig. 5. The liquid and size of heater have been chosen analogously to the works Kabov et al. (2001a, 2001b), where the deformation and the interface temperature of locally heated liquid film

\[
\frac{d^2 \psi (Y)}{dY^2} + \xi^2 \varepsilon Pe 2Y \psi (Y) = 0 \tag{41}
\]

\[
\frac{d \psi (Y)}{dY} (0) = 0 \tag{42}
\]

\[
\frac{d \psi (Y)}{dY} (1) + Bi \psi (1) = 0 \tag{43}
\]

$J_\nu (\xi)$ is the Bessel function of the first kind, coefficients $G_i$ are defined by formula:

\[
G_i = \frac{2\xi_i}{\left[ \frac{\partial \psi (1)}{\partial \xi} \right]_{\xi=\xi_i} - \psi (1) \left[ \frac{\partial^2 \psi (1)}{\partial Y \partial \xi} \right]_{\xi=\xi_i}}.
\]

Let us notice that the solution (37) takes place for any functions $\eta_i$ having an integrable distributional derivative.

**Fig. 5.** Temperature distribution on the film surface for different Reynolds numbers, 10% solution of ethyl alcohol in water, Bi=0.00366, L=0.0067 m.

**Fig. 6.** Temperature distribution on the film surface depending on Biot number, FC-72, $T_0=30^\circ\text{C}$, Re=4.9, $L=0.01$ m, $q_y=4\times10^{-4}$ m²/s, $\tau=0.02526$ kg/s²m
\[ U_x + V_y = 0 \]  
(46)

\[ \varepsilon Pe (U \theta_x + V \theta_y) = \theta_{yy} \]  
(47)

\[ U(X, 0) = V(X, 0) = 0 \]  
(48)

\[-\frac{\lambda}{\varepsilon} \theta_x(X, 0) = \varepsilon (\theta(X) - \theta(X - 1)) \]  
(49)

\[ P = -C^{-1} \vec{n}_{\text{xxx}} \]  
(50)

\[ U_x + \Sigma \vec{n}_x - T = 0 \]  
(51)

\[ \theta_x(X, \vec{n}(X)) = -B \varepsilon (\theta(X, \vec{n}(X)) - \theta) \]  
(52)

\[ \theta(-\infty, Y) = 0 \]  
(53)

\[ \vec{n}_x = \frac{V(X, \vec{n}(X))}{U(X, \vec{n}(X))} \]  
(54)

Where \( C = \frac{\mu}{\sigma} \varepsilon^{-3} \) and \( \text{Re}_e C = \mathcal{O}(1) \) at \( \varepsilon \to 0 \).

The equation of continuity (46) and kinematic condition (54) give

\[ \frac{\partial}{\partial X} \int_0^{\xi(X)} U(X, Y) dY = 0 \]  
(55)

The pressure out of (45) and boundary condition (50) is expressed as \( P = -C^{-1} \vec{n}_{\text{xxx}} + \rho gh \overline{n} \cos \varphi / p_0 \).

Using last expression the equation (44) can be written in the following form

\[ \varepsilon U_{yy} = -\varepsilon C^{-1} \vec{n}_{\text{xxx}} + \frac{\rho gh}{p_0} \cos \varphi \overline{n}_x - \frac{\rho gh}{p_0} \sin \varphi \]  
(56)

The integration of equation (56) gives the equation for film thickness

\[ \int_0^{\xi(X)} U dY = \overline{n}^2 \]  
(57)

\[ \frac{\overline{n}}{3} \left( \varepsilon C^{-1} \vec{n}_{\text{xxx}} + \frac{\rho gh}{p_0} \cos \varphi \overline{n}_x + \frac{\rho gh}{p_0} \sin \varphi \right) + \frac{\overline{n}}{2} \left( T - \Sigma \vec{n}_x \right) = \varepsilon \gamma \]  
where \( \gamma \) is dimensionless flow rate.

For horizontally located channel \( \gamma = \int_0^Y UdY = \overline{n}^2 \), and equation (57) becomes

\[ \overline{n} = \frac{1}{2} \Sigma \overline{n}_x \]  
(58)

Let \( \overline{n}(X) = 1 + h(X) \), where \( \|h\| \ll 1 \). Linearizing equation (57) we obtain

\[ h_{\text{xxx}} - \frac{\rho gh}{p_0} \cos \varphi h_x + 3C \left( T - \Sigma h - \frac{\rho gh}{p_0} \sin \varphi \right) h = 0 \]  
(59)

If thermocapillary tangential stress is much less than tangential stress caused by gas flow, \( |\Sigma \overline{n}_x| \ll |T| \), then neglecting the component \( \Sigma \overline{n}_x \) in left-hand member we come to the following linearized equation

\[ h_{\text{xxx}} - \frac{\rho gh}{p_0} \cos \varphi h_x + + 3C \left( T + \frac{\rho gh}{p_0} \sin \varphi \right) h = f(X) \]  
(60)

where \( f(X) = 3C \left( \gamma - \frac{1}{2} \left( T - \Sigma \overline{n}_x \right) \right) \frac{\rho gh}{\varepsilon p_0} C \sin \varphi \).

The thermocapillary force ought not be neglected in right-hand member since the flowrate and tangential stress can be comparable in values and the inclination angle can be close to zero.

The solution of equation (60) is presented as convolution of function \( F(X) \) and \( f \) (Vladimirov, 1981):

\[ h_1(X) = \int_{-\infty}^{\infty} F(X - \xi) f(\xi) d\xi \]  
(61)

Here \( F(X) \) is the fundamental solution of operator

\[ A(\frac{d}{dX}) = \frac{d^3}{dX^3} - \frac{C \rho gh}{p_0} \cos \varphi \frac{d}{dX} + 3C \left( T + \frac{\rho gh}{\varepsilon p_0} \sin \varphi \right) \]  

The characteristic polynomial of the operator \( A \) is

\[ s^3 - C \frac{\rho gh}{p_0} \cos \varphi s + 3C \left( T + \frac{\rho gh}{\varepsilon p_0} \sin \varphi \right) = 0 \]  

\[ s_1 = -\varepsilon, \ s_{2,3} = \frac{c}{2} \pm ib \]  
are the roots of the polynomial.

If the discriminant of the characteristic polynomial \( D > 0 \), then it has one real and two complex conjugated roots; if \( D = 0 \), then all three roots are real, and two of its are congruent; if \( D < 0 \), then all three roots are real and different. The fundamental solution has the following form (Marchuk and Kabov, 1998; Kuznetsov, 2000):

when \( D > 0 \)
where $\sin \varphi$, $\cos \varphi$, are the roots of $\lambda^2 + \left(\frac{3}{2\sigma_0 h_0} + \rho g \sin \varphi \right) = 0$. Let us denote $l_\tau = \sqrt{\frac{\sigma_\tau}{\rho g}}$ capillary length. We obtain, that the condition $D \leq 0$ equivalents to the condition

$$\tau \leq \frac{\rho h_0}{3 l_\tau} \left( \frac{\cos \varphi}{\rho g} \right)^2 - \sin \varphi.$$  

\[ \text{I.e. if the hydrostatic forces overbalance the surface forces, the damped oscillations of free surface will not exist.} \]

The thermocapillary forces the more possibility of the damped oscillations appearance.

The discriminant of characteristic polynomial is positive at the absence of gravity, and fundamental solution of equation (60) has the form of (62). In this case the damped oscillations always exist.

$$h_{xx} = \frac{\rho h_0}{p_0} h_t = \frac{3}{2} C\Sigma\bar{\rho} x$$

Integrating this equation we obtain the linear equation of second order.

$$h_{xx} - \frac{\rho h_0}{p_0} h_t = \frac{3}{2} C\Sigma\bar{\rho} x$$

The solution of (65) is presented as convolution

$$h_t(X) = \int_{-\infty}^{\infty} F(X - \xi) \frac{3}{2} C\Sigma\bar{\rho} d\xi,$$

where $F(X) = \frac{\delta l_\tau}{2 h_0} \left( \chi(-X) \exp \left( \frac{h_0}{\delta l_\tau} X \right) - \chi(X) \exp \left( -\frac{h_0}{\delta l_\tau} X \right) \right)$ is fundamental solution of corresponding homogeneous equation.

The calculations of the film temperature and corresponding deformations are executed. The calculated relative film thickness along the channel for different heat flux density and different Bi numbers is presented in Fig. 7. The temperature gradient in the heater area causes the thermocapillary tangential stress directed toward to the main flow and therefore the increasing of the film thickness is observed in the heating area. The free surface temperature is decreased outside of the heater (Fig. 5-6). The thermocapillary force is directed streamwise, therefore the decreasing of the film thickness relatively to the initial film thickness is observed. The calculation predicts the formation of thermocapillary thickening up to 30-50% compare with the initial film thickness at low heat transfer to the gas phase and relatively high heat flux density on the heater. A thermocapillary bump of analogous size of the order of magnitude has been observed in experiments Kabov et al. (2001a, 2001b) at local heating of falling liquid film under gravity along a vertical plate.

CONCLUSIONS

The calculations show that for horizontally located channel the velocity profile in the film is practically linear, it allows to make the conclusion that the gas friction is the main moving force for the film. The liquid motion mainly caused by gravity for inclined at an angle 5 degrees to the horizon channel. The velocity profile in that case is semiparabolic.

It is shown that the temperature on the film surface reaches the maximum beyond the border of the heater. The position of the maximum of temperature is replacing downstream with increasing Re number. When Biot number is growing up the maximum temperature on the film surface is going down.

The formation of a liquid bump caused by the thermocapillary effect in the region where thermal boundary layer reaches the film surface is obtained. It may exist damped oscillations of free surface before the bump up to flow according to the obtained solution for inclined channel. It depends on the forces balance in the film.

Fig. 7. Film deformation , FC-72, Re=4.9, L=0.01m, $\varphi=0$ degree, $q_g=4e-04$ m$^3$/s, $r=0.02526$ kg/s$^2$m, 1-Bi=0.005, $q=10000$ W/m$^2$, 2- Bi=0.005, $q=20000$ W/m$^2$, 3-Bi=1, $q=10000$ W/m$^2$, 4-Bi=1, $q=20000$ W/m$^2$. For horizontally located channel the linearized equation for (58) becomes

$$h_{xxx} - C\rho g h_0 h_t = \frac{3}{2} C\Sigma\bar{\rho} x$$

Integrating this equation we obtain the linear equation of second order.

$$h_{xx} - \frac{\rho h_0}{p_0} h_t = \frac{3}{2} C\Sigma\bar{\rho} x$$

The solution of (65) is presented as convolution

$$h_t(X) = \int_{-\infty}^{\infty} F(X - \xi) \frac{3}{2} C\Sigma\bar{\rho} d\xi,$$

where $F(X) = \frac{\delta l_\tau}{2 h_0} \left( \chi(-X) \exp \left( \frac{h_0}{\delta l_\tau} X \right) - \chi(X) \exp \left( -\frac{h_0}{\delta l_\tau} X \right) \right)$ is fundamental solution of corresponding homogeneous equation.

The calculations of the film temperature and corresponding deformations are executed. The calculated relative film thickness along the channel for different heat flux density and different Bi numbers is presented in Fig. 7. The temperature gradient in the heater area causes the thermocapillary tangential stress directed toward to the main flow and therefore the increasing of the film thickness is observed in the heating area. The free surface temperature is decreased outside of the heater (Fig. 5-6). The thermocapillary force is directed streamwise, therefore the decreasing of the film thickness relatively to the initial film thickness is observed. The calculation predicts the formation of thermocapillary thickening up to 30-50% compare with the initial film thickness at low heat transfer to the gas phase and relatively high heat flux density on the heater. A thermocapillary bump of analogous size of the order of magnitude has been observed in experiments Kabov et al. (2001a, 2001b) at local heating of falling liquid film under gravity along a vertical plate.

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oscillations of free surface do not exist for horizontally located channel.
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NOMENCLATURE

\( a \) temperature conductivity coefficient, m\(^2\)/s
\( B \) channel width, m
\( Bi \) Biot number, dimensionless
\( C \) analogue of capillary number, dimensionless
\( c_p \) specific heat capacity, J/kg\( ^\circ\)K
\( g \) gravity acceleration, m/s\(^2\)
\( H \) channel high, m
\( h \) film thickness, m
\( h_0 \) initial film thickness, m
\( h_t \) relative thickening, dimensionless
\( h_t \) film thickness, dimensionless
\( L \) heater length, m
\( l_c \) capillary length, m
\( P \) pressure, dimensionless
\( p \) pressure, Pa
\( p_a \) pressure of surround gas, Pa
\( p_0 \) characteristic pressure, Pa
\( Pe \) Peclet number
\( \dot{Q}_g \) gas flow rate, m\(^3\)/s
\( \dot{Q}_l \) liquid flow rate, m\(^3\)/s
\( q(x) \) heat flux density, W/m\(^2\)
\( q_l \) volumetric liquid flow rate, m\(^3\)/s
\( q_g \) volumetric gas flow rate, m\(^3\)/s
\( q_0 \) heat flux density, W/m\(^2\)
\( R \) curvature radius of the free surface, m
\( Re(l) \) liquid Reynolds number
\( Re(g) \) gas Reynolds number
\( Re \) film Reynolds number
\( T \) temperature of the film, K
\( T_a \) temperature of surround gas, K
\( T_0 \) initial temperature of the film, K
\( \Delta T \) characteristic temperature drop, K
\( \Gamma \) temperature on the film surface, K
\( T_t \) temperature of surround gas, dimensionless
\( T_f \) film surface temperature, dimensionless
\( \varepsilon \) ratio of linear film scales, dimensionless
\( \varphi \) inclination angle of the channel, degree
\( \rho, \rho_l \) liquid density, kg/m\(^3\)
\( \rho_g \) gas density, kg/m\(^3\)
\( \lambda \) thermal conductivity coefficient, W/mK
\( \mu, \mu_l \) liquid dynamic viscosity, kg/ms
\( \mu_g \) gas dynamic viscosity, kg/ms
\( \eta(X) \) finite function
\( \chi(X) \) Heaviside function
\( \tau \) tangential stress on the interface, dimensionless
\( \Sigma \) dimensionless surface tension
\( \sigma \) liquid surface tension coefficient, N/m
\( \sigma_0 \) liquid surface tension coefficient specified at temperature \( T_0 \), N/m
\( \sigma_r \) surface tension temperature dependence coefficient, N/mK
\( (\sigma'_a) \) viscous stress tensor

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