HEAT TRANSFER AND FILM DYNAMIC IN SHEAR-DRIVEN LIQUID FILM COOLING SYSTEM OF MICROELECTRONIC EQUIPMENT

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ABSTRACT

The conjugated two-dimensional model, based on long-wave theory, of a steady laminar flow of liquid film and co-current gas flow in planar channel with the height varied from 150 to 500 µm is performed. A chip with the several millimeters length is located on the bottom wall of channel. The linearized approximation of the problem is obtained analytically. Numerical calculations are executed for liquid FC-72 and Nitrogen gas flow. In contrast to a case of a large channel, there is essential an influence of liquid film deformations on pressure and velocity in a gas phase.

INTRODUCTION

The recent development of micro-systems is intimately linked with the problem of thermal regulation. Multiple chip modules (MCM’s), i.e. a multiple array of chips connected in a computer circuit, require ever higher rates of heat removal as computers are designed to have more processing ability per unit time. The levels of energy generation in high-speed computer chips are now approaching values so high (up to 150-200 W) as to exceed the capabilities of today’s air-cooling techniques. Accordingly, liquid single-phase micro-channel, two-phase flow and jet-spray evaporative cooling systems appear to be imperative (Schmidt, 2003). The local heat flux densities of these systems reach nowadays values up to 100-200 W/cm²). The electronic industry is ready to provide components where the average heat flux densities may reach values much higher than 100 W/cm².

A particularly promising technological solution allowing to reach high heat flux densities and to decrease a space and mass required for cooling hardware is a set-up where heat is transferred from MCM package to a very thin liquid film moving under friction of a forced gas or vapor flow in a micro-channel (shear-driven film evaporator, SDFE). Some investigations related to the application of a thin liquid film moving under friction of a forced gas flow in a micro-channel for cooling systems of microelectronic equipments have been reported in Sherwood and Cray (1992), Bar-Cohen et al. (1995). The authors of last paper have investigated experimentally a system with Nitrogen gas flow and liquid FC-72 in a symmetrically heated 0.508 mm wide channel.

Stability of joint flow of liquid film and gas is a complex problem that has not been fully studied up to now. The shear and normal stresses induced at the interface by the gas flow are responsible for film instabilities that yield nonlinear patterns (or waves), which are of crucial importance in the heat transfer process (Aktershev and Alekseenko, 1996).

Temperature gradient at the gas-liquid interface produces thermocapillary forces, which induce convection and intensive heat and mass transfer. The Marangoni effect has an essential influence on heat transfer intensity and may lead to film breakdown. In Kuznetsov (2000) the movement of a locally heated liquid film by co-current gas flow and gravity is investigated. In Gatapova et al. (2003) a two-dimensional model of a steady laminar flow of liquid film and co-current gas flow in a plane channel is considered. Gas flow influence on a liquid is simulated in both papers by a constant tangential stress on the interface that can be
justified only at absence of a liquid influence on a gas dynamics.

The objective of the present work is to study theoretically thin film hydrodynamics and thermal processes in SDFE. The theoretical approach considers simultaneously the motion of liquid layer and co-current gas flow, modeling film cooling system of single chip embedded in a substrate, taking into account convective heat transfer, surface tension and Marangoni effect.

![Diagram of a vapor shear-driven film evaporator.](image)

**Fig. 1 Vapor shear-driven film evaporator.**

**SHEAR-DRIVEN FILM EVAPORATOR**

The principle of SDFE is similar to heat transfer devices, such as Micro Heat Pipe and Micro Two Phase Loop with “natural” or forced circulation of liquid that developed for ground and space applications (Sarte et al., 2000; Delil, 2002). All these systems enable to extract the heat in very localized zones and also to transport it towards higher exchange surface areas where other technologies take the relay. The heat flux density evacuated reaches up to 200 W/cm² (Palm, 2003).

The principal technological objective of the present research is to increase the operating speeds of a new generation of electronic systems and computers using advanced thin films liquid cooling technology. Evaporation is a particularly efficient mean to transport energy, due to the high enthalpy difference (or latent heat) between vapor and liquid. Actually, the most efficient devices used to cool electronic components are based on evaporation and transport of the vapor or two-phase flow (mixture of liquid and vapor) towards a condenser, where the heat pumped by the evaporation can be released using other mechanisms (e.g. convection, radiation). Nevertheless, the maximal heat flux density that can be evacuated remains limited, which represents the main shortcoming for further performance improvement and miniaturization of electronic components.

This limit could be overcame either by improving the design of evaporator sections, or by better mastering the process of evaporation itself when length scales are such that a number of microscopic effects become important. This technological projection leads to new problems involving two-phase flows at micro-scales, where classical fluid mechanics might not remain valid. Indeed, the behavior of the flow is influenced by several small-scale phenomena (wetting, interface curvature, Van der Waals forces, non-equilibrium kinetic effects, …) when the channel hydraulic diameter decreases and evaporation of extremely thin liquid film takes place, and it is very often not possible to apply the classical laws for heat transfer that usually yield the critical heat flux. Determination of heat and mass transfer at such small scales are therefore an actual challenge.

Figure 1 shows a schematic of set-up where heat is transferred from MCM to a liquid film moving under friction of a forced vapor flow in a micro-channel (vapor shear-driven film evaporator, VSDFE). Figure 2 shows a schematic of set-up where heat is transferred from a single chip to a liquid film moving under friction of a forced noncondensable gas flow in a micro-channel (gas shear-driven film evaporator, GSDFE).

![Diagram of a gas shear-driven film evaporator.](image)

**Fig. 2 Gas shear-driven film evaporator.**

**GEOMETRY AND LIQUIDS**

The two-dimensional model of a steady laminar flow of liquid film FC-72 and co-current Nitrogen gas flow in plane channel with the height varied from 150 µm to 500 µm is considered. The length of the heater varied for 2.5 to 5 mm. Initial film thickness varied from 50 µm to 200 µm and initial velocity of gas varied from 2 m/s to 5 m/s. The values of the Nitrogen gas properties at the initial temperature 20 °C and atmospheric pressure are: \( \rho_T = 1.2505 \text{ kg/m}^3, \mu_T = 1.775 \times 10^{-5} \text{ Pa s}, \gamma = 1.401.\)

This dielectric liquid is widely used in cooling systems of microelectronic equipment. The surface tension is assumed to depend linearly on temperature: \( \sigma(T) = \sigma_0 - \sigma_f(T - T_0), \quad \sigma_0, \sigma_f = \text{const > 0}. \) The values of the liquid properties at the initial fluid temperature 20 °C are: \( \rho = 1687.8 \text{ kg/m}^3, \kappa = 0.058 \text{ W/(m K)}, \quad \sigma_0 = 12.57 \times 10^3 \text{ N/m}, \quad \sigma_f = 1.14 \times 10^4 \text{ N/(m K)}, \quad c_p = 1046 \text{ J/kg K}, \quad \text{Pr} = 13.3. \) It is assumed that these properties do not depend on temperature.

At the temperature range from \( T_f = 20 \text{ °C} \) to \( 40 \text{ °C} \) the dynamic viscosity is reduced from \( \mu = 0.743 \times 10^3 \text{ Pa s} \) to \( \mu = 0.572 \times 10^3 \text{ Pa s}. \) The 29.9 % variation of viscosity takes place and should be taken into account. We choose a polynomial dependence based on existing data as follows

\[
\mu(T) = 1.01202E-03 - 1.83076E-05 \times T + 2.03928E-07 \times T^2 + 5.48397E-10 \times T^3 + 5.39932E-11 \times T^4 - 1.59905E-12 \times T^5 - 1.11789E-14 \times T^6 \text{ Pa s}. \quad (1)
\]
PROBLEM FORMULATION

Geometry of the problem is shown in Fig. 2. We choose a Cartesian coordinate system \((x, y)\) so that the axis \(Oy\) is orthogonal to the substrate, and the axis \(Ox\) is directed along the liquid flow. Let the liquid occupy a domain \(-\infty < x < \infty, 0 < y < h(x)\), where the motion of the liquid is described by the following Navier-Stokes, continuity and energy equations:

\[
\rho \left( \bar{v} \cdot \nabla \right) \bar{v} = \bar{g} - \nabla p + 2 \text{div}(\mu D) \tag{2}
\]

\[
\nabla \cdot \bar{v} = 0, \quad \bar{v} \cdot \nabla T = \chi \Delta T. \tag{3}
\]

The boundary conditions have the form: at the wall \((y=0)\)

\[
\bar{v} = 0, \tag{4}
\]

\[
\kappa T_y - h_1 (T - T_0) = -q_1, \tag{5}
\]

at the interface \((y=h(x))\)

\[
u h' - v = 0, \tag{6}
\]

\[
p - p_g = 2\sigma K + 2\mu \bar{n} \cdot \bar{n}, \tag{7}
\]

\[
2\mu \bar{s} \cdot \bar{n} = \nabla \sigma + \tau, \tag{8}
\]

\[
\kappa \frac{\partial T}{\partial n} + h_2 (T - T_0) = 0. \tag{9}
\]

Gas composition and gas temperature are supposed to be constant. It means that evaporation and heat transfer on the interface have not been taken into account for gas flow. Boundary layer in the gas \(\delta_g\), where viscous forces are important, is supposed to be small because the viscosity of the gas is small and the velocity of the gas is large in comparison with film one. Gas is ideal and tangential stress on the interface is determined as

\[
\tau = 2\mu_g \frac{\partial U}{\partial n} \bigg|_{y=h} \approx \Lambda U, \quad \Lambda \approx \frac{2\mu_g}{\delta_g}. \tag{10}
\]

It is supposed that gas is polytrophic. Law of mass conservation has the form:

\[
\rho_g U (H - h) = \rho_0 U_0 (H - h_0), \tag{11}
\]

and Bernoulli’s law is:

\[
\frac{U^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p}{\rho_0} = \frac{U_0^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0}. \tag{12}
\]

The adiabatic condition leads to:

\[
\frac{p_g}{p_0} = \left( \frac{\rho_g}{\rho_0} \right)^{\gamma/\gamma - 1}. \tag{13}
\]

Algebraic Eq. (14) for gas velocity is obtained from Eqs. (11)-(13):

\[
\frac{U^2}{2} + \frac{\gamma}{\gamma - 1} \frac{U_0}{U} \left( \frac{U_0}{p_0} \right)^{-\frac{\gamma - 1}{\gamma}} = \frac{U_0^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p_0}{\rho_0}. \tag{14}
\]

Gas pressure is described by the formula:

\[
p_g = p_0 \left[ 1 + \frac{\rho_0 (U_0^2 - U^2)(\gamma - 1)}{2\gamma \rho_0} \right]^{\frac{\gamma - 1}{\gamma}} \tag{15}
\]

In case of isothermal and nondeformable liquid film, i.e. \(h = h_0\) and \(U = U_0\), the problem can be solved precisely in a form \(u_0 = u_0(y)\), \(v_0 \equiv 0\), \(T \equiv T_0\). Solution of Eqs. (2)-8 at such conditions leads to:

\[
u_0 = \frac{\tau_0}{\mu_0} y, \quad Q = \frac{\tau_0 h_0}{2\mu_0}. \tag{16}
\]

It is known that there is an approximate correlation of the film interface velocity and the average gas velocity in the form (Kuznetsov, 2000):

\[
u_s = k U_0, \quad k = \left( \frac{\mu_0 / \rho_0}{\mu / \rho} \right)^{\gamma/\gamma - 1} \tag{17}
\]

Equations (16), (17) yield to the formula for tangential stress on the interface:

\[
\tau_s = k \frac{\mu_0 U_0}{h_0}. \tag{18}
\]

It is supposed that in case of deformable film Eq. (18) is also valid:

\[
\tau = k \frac{\mu h_0}{h} U. \tag{19}
\]

Scales of velocity, length and temperature are assigned as:

\[
U_* = \frac{\mu_0}{\rho h_0}, \quad l = \left( \frac{\sigma h_0}{\rho U_*} \right)^{\gamma/\gamma - 1}, \quad [T] = \frac{h_0 [q]}{\kappa}. \tag{20}
\]
where \( q = \text{Sup}_x q_i(x) \). Variables \( x, y, u, v, T \) is changed by \( x, \xi, u', v', T \), where \( \xi = y/h(x), u' = u(h(x), v' = v-u \xi h(x) \). In new variables the domain of motion is fixed \( 0 < \xi < 1 \). Continuity equation has the same form and kinematic condition (6) is simplified. The non-dimensional variables \( \hat{x}, \xi, \hat{u}, \hat{v}, \hat{h}, \hat{T}, \hat{\theta}, \hat{\mu}, \hat{q_i}, \hat{\rho_s} \) are defined as:

\[
\begin{align*}
\hat{x} = & \xi, \quad \hat{u}_i = U_i h, \quad \hat{v} = U_j h, \quad \hat{h} = \hat{\mu} \hat{h}, \\
p = & p_0 + \sigma_0 h_0 \hat{p}/l^2 - \rho g h_0 \hat{\xi} \hat{h}, \quad T = T_0 + \hat{T} \theta, \\
U = & U \hat{h}, \quad q_i = \left[ q \right] \hat{q_i}, \quad \rho_s = \rho_{h0} \hat{p}_s/l^2.
\end{align*}
\]

(21)

Assuming the long-wave approximation to be valid in terms of the small parameter \( \varepsilon = h_0/l \ll 1 \), we write below the system of Eqs. (2)-(8) with the non-dimensional variables at the zero-order approximation with respect to \( \varepsilon \) (symbols " ^ " are omitted)

\[
\begin{align*}
\left( \mu \xi \right)_{\xi} - h^3 p_s = & 0, \\
p_s = & 0, \\
D(u\xi_{\xi} + v\xi_{\xi}) = & \frac{1}{h} \theta_{\xi}, \\
\xi_{\xi} + \nu_{\xi} = & 0,
\end{align*}
\]

(22)

(23)

(24)

(25)

with the boundary conditions at the wall (\( \xi = 0 \))

\[
\begin{align*}
u = & 0, \quad v = 0, \\
\frac{1}{h} \theta_{\xi} - B_i \theta = & -q_i,
\end{align*}
\]

(26)

(27)

and at the free surface (\( \xi = 1 \))

\[
\begin{align*}
u = & 0, \\
-\rho = & h^m - Ah - p_g,
\end{align*}
\]

(28)

(29)

\[
\mu \xi_{\xi} = -M a h \theta_{\xi} + k_i h^2U,
\]

(30)

\[
\frac{1}{h} \theta_{\xi} + B_i \theta = 0
\]

(31)

Equation (14) in non-dimensional form is

\[
U^2 + E \left( \frac{\beta - 1}{\beta - h} \right) = 1 + E.
\]

(32)

Here the dimensionless criteria of similarity are defined as

\[
A = \frac{g h_0^2}{l U^2}, \quad D = \frac{h_0 \mu c_p}{l k}, \quad Ma = \frac{\sigma_T h_0}{\mu d U}, \quad k_i = k U_0 / U.
\]

(33)

Let us determine functions \( F(x, \xi), G(x, \xi), \Phi(x, \xi), \Gamma(x, \xi), \varphi(x), \delta(x) \) as follows:

\[
\begin{align*}
F(x, \xi) = & \int_0^\xi \frac{1 - \lambda}{\mu(\theta(x, \lambda))} d\lambda, \\
G(x, \xi) = & \int_0^\xi \frac{1}{\mu(\theta(x, \lambda))} d\lambda, \\
\Phi(x, \xi) = & \int_0^\xi F(x, \lambda) d\lambda, \\
\Gamma(x, \xi) = & \int_0^\xi G(x, \lambda) d\lambda, \quad \phi = \Phi|_{\xi=1}, \quad \delta = \Gamma|_{\xi=1}.
\end{align*}
\]

(34)

It follows from Eq. (23) that \( p = p(x) \), therefore using Eq. (29) we can conclude that Eq. (29) is valid in the whole domain. Integrating Eq. (22) from \( \xi = 0 \) and using condition (30) we get

\[
-Mah^2 \theta_{\xi} + k_i h^2 \hat{U} - \mu \xi_{\xi} + h^3 \left( h^m - A h - p_g \right) = 0,
\]

(35)

where \( \theta = \theta(x, 1) \). Dividing Eq. (35) by \( \mu \) and integrating it from 0 to \( \xi \) using Eqs. (26) and (34) yields:

\[
u = h^3 F \left( h^m - A h - p_g \right) - Mah^2 G \theta_{\xi} + k_i h^2 GU.
\]

(36)

Integrating continuity Eq. (25) from 0 to \( \xi \) with taking into account Eqs. (26) and (36) leads to:

\[
\begin{align*}
v = & - \left[ h^3 \Phi \left( h^m - A h - p_g \right) - Mah^2 \Gamma \theta_{\xi} + k_i h^2 \Gamma U \right] \theta_{\xi},
\end{align*}
\]

(37)

Substituting Eq. (37) into the boundary condition (28) we obtain an Eq. for the film thickness in the following form

\[
\begin{align*}
\frac{1}{h} \phi \left( h^m - A h - p_g \right) - Mah^2 \delta \theta_{\xi} + k_i h^2 \delta U = & \text{const} = k_s \delta_s
\end{align*}
\]

(38)

Here \( \delta_s = \lim_{x \to \pm \infty} \delta(x) \). Last equality in Eq. (38) is satisfied because at the local heating the following limiting conditions are supposed to be valid:

\[
\begin{align*}
h \to 1, \quad U \to 1, \quad \theta \to 0, \quad p_g \to 0 \quad \text{at} \quad x \to \pm \infty.
\end{align*}
\]

(39)
The problem is reduced to finding of two functions \( h(x) \) and \( \theta(x, \xi) \), from the system of Eqs. (24), (38) with boundary conditions (27), (31), (39). Gas velocity \( U(x) \) is determined as a solution of equation (32) using initial data and values of unknown functions \( h \), \( \theta \). Functions \( u \), \( v \), \( \varphi \), \( \delta \), \( p_g \) are assigned by the explicit formulas.

**SOLUTION OF LINEARIZED EQUATION**

Let us write the problem in the linear form assuming that \( h(x) = 1 + z(x) \), where \( z(x) \) is a new dimensionless variable function (\( |z(x)| << 1 \)). Assuming that the intensity of the heating is small i.e. \( Ma << 1 \) and \( |z'| |, |z''| << 1 \), that means small deformations of the film surface and small curvature of deformations. Let us present gas velocity as \( U = 1 + a_1 z + a_2 z^2 + \ldots \) that leads to the terms in Eq. (32):

\[
U^2 = 1 + 2a_1 z + o(z),
\]

\[
\left( \frac{\beta - 1}{\beta - \delta} \right) = 1 + \frac{\gamma - 1}{\beta - 1} z + o(z),
\]

\[
\frac{1}{U^{-1}} = 1 - (\gamma - 1) a_1 z + o(z).
\]

Substituting these formulas into Eq. (32) and neglecting all nonlinear \( z \) terms we obtain:

\[
a_i = \frac{E(\gamma - 1)}{[E(\gamma - 1) - 2](\beta - 1)}. 
\]

Here \( E >> 1 \). For example, for Nitrogen at \( p_0 = 1 \) bar, \( T_0 = 20 ^\circ \text{C} \) and \( U_0 = 1 \) m/s we have \( E \approx 10^6 \) that leads to \( a_i \approx (\beta - 1)^{-1} \). From Eqs. (15) and (21) we obtain:

\[
p_g = \frac{1}{2} N \left( 1 - U^2 \right), N = \frac{\rho_0 U_0^2 l^2}{\sigma_0 h_0},
\]

\[
U = 1 + \frac{z}{\beta - 1}, p_g = -N \frac{z}{\beta - 1}.
\]

Let us simplify functions \( \varphi \) and \( \delta \) in Eq. (38). It is supposed that if \( \theta_1(x) = 0(x, 0), \theta_2(x) = 0(x, 1), \) and \( \mu_1(x) = \mu(\theta_1(x)), \mu_2(x) = \mu(\theta_2(x)) \) then \( \mu = \mu_1 + \xi (\mu_2 - \mu_1) \).

Explicit functions \( F, G, \Phi, \Gamma \) are calculated from formulas (34). For functions \( \varphi, \delta \) we obtain formulas

\[
\varphi = \frac{1}{\mu_1} \left[ \frac{1}{2 \alpha} + \frac{1 + \alpha}{\alpha} \left( (1 + \alpha) \ln(1 + \alpha) - \alpha \right) \right],
\]

\[
\delta = \frac{1}{\mu_1 \alpha} \left[ ( \alpha + 1 ) \ln(1 + \alpha) - \alpha \right].
\]

Here \( \alpha = (\mu_2 - \mu_1)/\mu_1 \). If \( \alpha \) is small, then using \( \ln(1 + d) = d - d^2/2 + d^3/3 - \ldots \) we get:

\[
\varphi = \frac{1}{\mu_1} \left[ \frac{1}{3} - \frac{\alpha}{6} + o(\alpha),
\]

\[
\delta = \frac{1}{\mu_1} \left[ \frac{1}{2} - \frac{\alpha}{6} + o(\alpha) \right].
\]

According to Eq. (39) all disturbances are damped at large distance from the heating element that leads to \( \varphi_o = 1/3, \delta_o = 1/2 \). Also next Eqs. are valid:

\[
\varphi = \frac{3}{2} + \frac{\alpha}{2} + o(\alpha),
\]

\[
\delta = \frac{3}{2} + \frac{\alpha}{2} + o(\alpha).
\]

Let us divide Eq. (38) by \( \phi \) and substitute there Eqs. \( h = 1 + z, h^2 = 1 + 2z + o(z), h^3 = 1 + 3z + o(z) \), (49) and (44). At the first-order approximation with respect to small parameters (\( |z|, |z'|, |z''|, \alpha, Ma \)) we obtain:

\[
z'' = A_1 z + C z = \Pi,
\]

where

\[
C = 3k_1 \left( 1 + \frac{1}{2(\beta - 1)} \right), A_1 = \left( \frac{A - N}{\beta - 1} \right),
\]

\[
\Pi = \frac{3}{2} Ma \theta z + k_1 \left[ -\frac{\alpha}{2} + \frac{3}{2}(\mu_1 - 1) + \frac{3}{4} \mu_1 \alpha \right].
\]

Boundary conditions for equation (51) are

\[
\lim_{z \to \pm \infty} z(x) = 0.
\]

Problem (51), (53) can be solved exactly. Characteristic polynomial of Eq. (51) can be written as:

\[
\lambda^3 - A_1 \lambda + C = 0, A_1 = A - N/\beta - 1
\]

There is one real negative root \( \lambda_1 = -\alpha < 0 \), because left side of Eq. (54) is positive at \( \lambda = 0 \), and negative at \( \lambda \to -\infty \). If \( \lambda_2, \lambda_3 \) are also roots of Eq. (54) we get:

\[
\lambda^3 - A_1 \lambda + C = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3),
\]

or
\[ \lambda_1, \lambda_2, \lambda_3 = -C, \lambda_1 + \lambda_2 + \lambda_3 = 0, \text{or} \lambda_2, \lambda_3 = C/a, \lambda_2 + \lambda_3 = a. \] 
By the Viete theorem \( \lambda_2, \lambda_3 \) are roots of Eq. 
\[ \lambda^2 - a\lambda + C/a = 0, \text{therefore} \lambda_{2/3} = a/2 \pm \sqrt{a^2/4 - C/a}. \]

There are three cases: i) \( a^2/4 - C/a > 0 \), ii) \( a^2/4 - C/a = 0 \) and iii) \( a^2/4 - C/a < 0 \). It is known, that case i) is satisfied when inequality is valid:
\[ -A^4/4 + C^2/27 > 0, \quad (55) \]
case ii) is satisfied if (55) turns into equality and case iii) is satisfied if (55) is not valid. In the case i) \( \lambda_2 \neq \lambda_3 \) and \( \lambda_2, \lambda_3 > 0 \). Fundamental solution of Eqs. (51), (53) are
\[ \Omega(x) = \frac{e^{-ax}}{(\lambda_1 - \lambda_2)(\lambda_1 - \lambda_3)} \chi(x) - \left[ \frac{e^{ax}}{(\lambda_2 - \lambda_1)(\lambda_2 - \lambda_3)} \right] \chi(-x). \quad (56) \]
Case ii) leads to \( \lambda_2 = \lambda_3 = a/2 > 0 \) and
\[ \Omega(x) = \frac{4e^{-ax}}{a^2} \chi(x) + e^{ax/2} \left( \frac{4}{a^2} - \frac{2a}{a} \right) \chi(-x), \quad (57) \]
and in case iii) we get \( \lambda_{2/3} = a/2 \pm ib, \) \( b = \sqrt{C/a - a^2/4}, \)
\[ \Omega(x) = \frac{4}{9a^2 + 4b^2} e^{-ax} \chi(x) + \frac{4}{9a^2 + 4b^2} e^{ax/2} \chi(-x) \left[ \cos(bx) - \frac{3a}{2b} \sin(bx) \right]. \quad (58) \]
Solution of the problem (51), (53) has a form
\[ z(x) = \int_{-\infty}^{\infty} \Pi(\lambda) \Omega(x - \lambda) d\lambda. \quad (59) \]

Hence, if temperatures \( \theta_1 \) and \( \theta_2 \) are known the solution of linear problem can be constructed in quadratures. It is not necessary to solve the heat transfer problem in case of first type heat transfer conditions, when \( \theta_1 \) and \( \theta_2 \) are given instead of Eqs. (27) and (31).

Let us obtain exact solution of the heat transfer problem in case of third type of heat transfer conditions. We substitute expression \( 1/h = 1 - z + o(z) \) in Eq. (25) with boundary conditions (27) and (31). As components of modified velocity \( u \) and \( v \) we use averaged across the flow values in case of nondeformable liquid film i.e. \( u = v = 0 \). Thus, we get a simplified problem:
\[ D_1 \theta_x = \theta_x, -\infty < x < \infty, 0 < \xi < 1, \quad (60) \]
\[ \theta_x - B_i \theta = -q_1, \quad \xi = 0, \quad (61) \]
\[ \theta_x + B_2 \theta = 0, \quad \xi = 1. \quad (62) \]

Where \( D_1 = Dk_1/2 \). Let us apply Fourier transformation to Eqs. (60)-(62). Denoting
\[ \{ f(t, \xi) \} = \int_{-\infty}^{\infty} f(x, \xi) e^{ixt} dx. \quad (63) \]
we get:
\[ -itD_1 \{ \theta \} = \{ \theta \}, -\infty < t < \infty, \quad 0 < \xi < 1, \quad (64) \]
\[ \{ \theta \}_{\xi} - B_i \{ \theta \} = -\{ q_1 \}, \quad \xi = 0, \quad (65) \]
\[ \{ \theta \}_{\xi} + B_2 \{ \theta \} = 0, \xi = 1. \quad (66) \]
Solution of the boundary problem (64)-(66) has a form
\[ \{ \theta(t, \xi) \} = \{ q_1(t, \xi) \} \left[ \frac{e^{\alpha(1-i)}(\omega - B_2) + e^{\alpha(1+i)}(\omega + B_2)}{e^{\alpha - e^{-\alpha}} - e^{\alpha - \alpha}} \right] \quad (67) \]
where \( \omega = (1-i)\sqrt{\sqrt{D}/2} \). If \( B_1 = B_2 = 0 \), than we get
\[ \{ \theta(t, \xi) \} = \frac{2\{ q_1(t, \xi) \}}{\omega(e^{-\alpha} + e^{\alpha})}. \quad (68) \]
Solution \( \theta(x, \xi) \) can be obtained from the equation
\[ \theta(x, \xi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{ \theta(t, \xi) \} e^{ixt} dt. \quad (69) \]

**NUMERICAL CALCULATIONS AND DISCUSSION**

Calculations are carried out with the use of following iterative algorithm. Initially it is assumed that \( n=1, U=1, p_1=0, \theta=0 \). Functions \( F, G, \Phi, \Gamma, \phi, \delta \) are found from Eqs. (34). Than components of the modified velocity vector \( u, v \) are found from Eqs. (36), (37) and the heat transfer problem (24), (27), (31) is solved. Once \( \theta \) is known, new values of functions \( F, G, \Phi, \Gamma, \phi, \delta \) are found and a new value of function \( h(x) \) is determined from the solution of Eq. (38) with boundary conditions (39). Finally, values of velocity \( U(x) \) and pressure distribution \( p_d \) are found from the solution of Eq. (32). Then it is initiated the next iteration step. The iterative process is rapidly convergent. It is sufficient, in most cases, 20-50 iterations to achieve an accuracy \( 10^{-7} \). All calculation in the present work are performed for \( B_1=B_2=0 \).
Fig. 3 shows a film deformation along the channel. The positive temperature gradient on the film surface in the heater area causes the thermocapillary tangential stress directed opposite to the gas flow and therefore the increasing of the film thickness is observed in the heating area (bump). The damped perturbations of free surface before the bump up to flow exist for all calculations. Calculations is performed for three cases: 1) gas flow parameters are constant, 2) gas pressure is constant and tangential stress depends on film deformation, 3) gas pressure and tangential stress depend on film deformation. Tangential stresses on the interface caused by the gas flow have a significant effect on the film deformation. In contrast gas pressure effect is insignificant.

The relative film thickness along the channel for different channel height and different heat flux are presented in Figs. 4, 5. Gas velocity increases with increase of film thickness deformation and this effect is more pronounced for small channel height. Increasing of tangential stresses on the interface caused by the gas flow decreases the film thickness. Under constant Re number with heat flux increasing, that correspond to thermocapillary effect increasing, the film deformation increasing also. The decreasing of the film thickness observed downstream the heater can be attributed to the temperature dependent viscosity effect.

The free surface and wall temperature as well as gas velocity and gas pressure are shown in Figs. 6-8 for the same flow parameters as in Fig. 5. In contrast to a case of infinite flow or a large channel height, there is essential an influence of liquid film deformations on pressure and velocity in a gas phase for co-current gas-liquid flow in a microchannel. The calculation predicts the formation of thermocapillary thickening up to 25% compare with the initial film thickness. Calculations are performed for FC-72 highly subcooled up to the saturation temperature (56 °C at
atmospheric pressure) and relatively small heat fluxes. Temperature of liquid does not exceed 55 °C because evaporation has not been taken into account in the present model, i.e. calculations show performance of the gas shear-driven film evaporator to cool a single chip by liquid convection only and not by evaporation and heat transfer to the gas phase.

**Fig. 7** Heat flux effect on gas velocity, $L=2.5$ mm, $h_l=50$ μm, $H=200$μm, $U_0=3$ m/s. 1- $q=0.2$ W/cm$^2$; 2- $q=0.5$; 3- $q=1.0$; 4- $q=1.50$; 5- $q=1.75$.

**Fig. 8** Heat flux effect on gas pressure, $h_l=50$ μm, $H=200$μm, $U_0=3$ m/s. 1- $q=0.2$ W/cm$^2$; 2- $q=0.5$; 3- $q=1.0$; 4- $q=1.50$; 5- $q=1.75$.

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**NOMENCLATURE**

- $h$ film thickness, m
- $Ma$ Marangoni number, dimensionless
- $P$ pressure, N/m$^2$
- $Q$ specific liquid flow rate, m$^3$/s
- $q_i$ heat flux on the wall of the heater, W/m$^2$
- $Re$ film Reynolds number $=Qd/u$, dimensionless
- $U$ gas velocity in $x$-axis, m/s
- $u,v$ liquid velocity components in $x$-, $y$-axis, m/s
- $Pr$ Prandtl number, dimensionless
- $T$ temperature, °C

**Greek symbols**

- $\Theta$ dimensionless temperature of the film
- $\rho$ density, kg/m$^3$
- $\kappa$ liquid thermal conductivity coefficient, W/(m K)
- $\mu$ liquid dynamic viscosity, kg/(m s)
- $\sigma$ surface tension, N/m
- $\gamma$ polytropic exponent, dimensionless
- $\tau$ tangential stress on the interface, N/$m^2$
- $\chi$ thermal diffusivity, m$^2$/s
- $\chi(-x)$ Heaviside function

**Subscripts**

- $0$ initial parameters of the flow (at $T=T_0$)
- $1,2$ value on substrate and gas-liquid interface
- $g$ gas
- $s$ surface
- $',, x,y, \xi, T$ derivations on $x$, $y$, $\xi$ and $T$

**REFERENCES**


