REGULAR STRUCTURES AND THERMOCAPILLARY REVERSE FLOW IN FALLING LIQUID FILM UNDER LOCAL HEATING

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ABSTRACT An experimental study of flow of a liquid film over the vertical and inclined (at an angle of 4 degrees with horizon) plate with the local heat source has been made. The temperature of the film surface has been measured by means of infrared thermography. Working liquid was 25% mixture of ethyl alcohol. Numerical calculations of film surface shape and streamlines have been made based on experimentally measured temperature distributions on the film surface. It has been established that horizontal roller formed at the upper edge of the heater has a thermocapillary nature and that there is a reverse flow in this roller.

INTRODUCTION

Thin liquid films flowing down under the gravity are widely used in different apparatuses of chemical technology, food, pharmaceutical and cryogenic industry, because they provide high intensity of heat and mass transfer. The Marangoni effect, caused by the surface tension gradient, in several cases, essentially influences the fluid flow, generating waves and forming vortex motion. The main objectives of this investigation are to measure temperature field on the film surface, to base on these measurements the hypothesis about thermocapillary nature of regular structures revealed by the authors in previous investigations, and to study the formation mechanism of these structures.

EXPERIMENTAL METHOD

The scheme of the experiment is shown in Fig. 1. The experimental technique is described in [1]. A 25% mixture of ethyl alcohol in water was used as a working liquid. The film was flowing down the plane vertical or inclined plate under the action of gravity. The surface of the plate was polished. The experiments were carried out in stationary conditions. The test section was opened into the atmosphere. The distance between the nozzle and the heat source was 43 mm and it was chosen so that, on the one hand, the film have stabilised velocity profile and thickness when it reaches the heater and, on the other hand, the heating units be situated in the region of smooth, waveless zone of film flow. Rectangular heat source of 6,5×13 mm simultaneously served as the detector of the heat flux. The temperature distribution on the liquid film surface was measured with infrared scanner “Sova-2”. The initial temperature of the film was measured by thermocouples and in this investigation it was 17 or 30°C. Temperature stabilizer allowed to maintain the temperature of the plate surface equal to the initial temperature of the film.
EXPERIMENTAL RESULTS

From the upper edge of the heater a heat boundary layer forms (Fig. 1) and at low Re numbers (Re<2) its length is essentially smaller than the heater one. After the temperature boundary layer emerges on the film surface there appears the longitudinal temperature gradient at the interface, that causes the thermocapillary tangential force. The thermocapillary force is directed from the regions with the higher surface temperature to the regions with the lower surface temperature ($\partial \sigma / \partial T < 0$).

During the experimental procedure at constant Re number and constant initial temperature of the liquid the heat flux density was stepwise increased. Regular horseshoe-like structures are formed in the film above the heater at the threshold value of heat flux density.

Fig 2 a) and 3 a) show the thermograms of the film surface with the regular structures. The structures consist of a horizontal liquid roller, rivulets flowing from it with a certain wavelength and thin film between rivulets. The roller, as a rule, places in the region of the upper edge of the heater. The flow of liquid with the structures is quasistationary that is the structures exist long time without remarkable changing. Temperature distribution on the film surface above the heater is substantially nonuniform both along the flow and transvers to the flow. Thin liquid film between the rivulets has the highest temperature. In the rivulets the temperature of the liquid is substantially lower then that in the region of the smooth film. Fig 2 b) and 3 b) represent absolute value of surface temperature gradient calculated using the film thermograms. Near the upper edge of the heater there is a region with the highest temperature gradient and this region coincides with the location of the horizontal liquid roller. Value of $\text{grad} T$ necessary for onset of structure formation growth with increasing Re number and with increasing angle of plate inclination.
Fig 2. Temperature and temperature gradient module distributions on liquid film surface, Re=2, $q=4.8$ W/cm$^2$, $T_0=30$ °C, $\Theta=90^\circ$.

Fig 3. Temperature and temperature gradient module distributions on liquid film surface, Re=3, $q=9.5$ W/cm$^2$, $T_0=30$ °C, $\Theta=90^\circ$.

THERMOCAPILLARY REVERSE FLOW IN HORIZONTAL ROLL

In [2] possible physical mechanism of regular structure formation is proposed which is based on the presence of surface temperature gradient and reverse flow of liquid in the roller. In [1] it is supposed that appearance of rivulets flowing from the horizontal roller is caused by instability of the roller. Further investigations showed that mechanism of rivulets formation has gravitationally-capillary nature [3]. In present work the process of horizontal roller formation and character of its instability are not considered. The main attention is paid to analysis of stationary two-dimensional flow of liquid in the roller after it has been formed. At given regime parameters (Re, $q$, $T_0$) the distribution of tangential stresses on interface is known from experiment. This approach allows to perform numerical calculations of the flow velocity of liquid and deformation of the interface without consideration of problem about heat spreading in liquid.

The system of equations for the film flowing (along the $x$-axis) under the gravity down the infinite plane surface, inclined at angle $\Theta$ with respect to the horizon, under the condition that only the surface tension coefficient depends on the temperature

$$\sigma = \sigma_0 + \sigma_T(T - T_0)$$

within approximation of thin liquid layer for the Re numbers of order 1 and in the suggestion that the film thickness weakly changes along the flow ($h_x^2 \ll 1$), looks as follows [4]
\[-\sigma_0 h_{xxx} + \rho g \cos \Theta h_x - \rho g \sin \Theta = \mu u_{yy} \]  

(2)

\[u_x + v_y = 0 \]  

(3)

With the boundary conditions:

\[u(x,0) = v(x,0) = 0 \]  

(4)

\[\mu u_y(x, h(x)) = \sigma_f T_x \]  

(5)

\[h(x) \int_0^1 \rho u(x, y) dy = \tilde{A} \]  

(6)

From this system we can get the following equation for the film thickness:

\[-\frac{\sigma_0}{\rho g} h_{xxx} - \sin \Theta \cos \Theta h_x = \frac{3\sigma_f T_x}{2h\rho} - \sin \Theta \frac{h_0^3}{\rho^3} \]  

(7)

We can make Eq. (7) dimensionless using \( l_o \) as characteristic length and \( \Delta T = T_{\text{max}} - T_0 \) as characteristic temperature. Then in operator form it can be represented as

\[ A\tilde{h} = f \]  

(8)

where operator \( A : H \rightarrow C_0^\infty (R) \), is defined as

\[ A\tilde{h} = -\tilde{h}''\tilde{h} - \sin \Theta \tilde{h} + \cos \Theta \tilde{h} \tilde{h} + \sin \Theta h_0^3 \tilde{h}^2 \]  

(9)

the set \( H = C_0^\infty (R) + \tilde{h}_0, f = (3\Delta T \sigma_f / 2\sigma_0)T' \)

From the equation for the velocity \( u(x, y) \) derived in [4] and from the equation of continuity we obtain stream function that is expressed through the film thickness derived from (8):

\[ \psi(x, y) = \frac{1}{3} \left( \frac{3\sigma_f T_x}{4\mu h} - \frac{3\tilde{A}}{2h^3} \right) y^3 + \frac{1}{2} \left( \frac{3\tilde{A}}{\rho h^2} - \frac{\sigma_f T_x}{2\mu} \right) y^2 \]  

(10)

Eq. (8) was solved by minimization of residual functional

\[ J(h) = \frac{1}{2} ||Ah - f||^2_{L^2} \]  

(11)

using the conjugate gradient method [5]. In calculation we used the temperature gradient on the film surface \( T_x \) measured by thermography.

The gradient \( J' h \) of the functional \( J(h) \) was determined by the formula [5]:

\[ J' h = (A_h)^* (Ah - f) \]  

(12)
where $A'_h$ is the Frechet derivative of the operator $A$ in the point $h$ , $(A'_h)^*$ is conjugate operator for $A'_h$. The element $-J'h$ shows in space $L_2(R)$ the most rapid decrease direction of the functional $J(h)$. The minimizing succession of the conjugate gradient method has the form:

$$h_{n+1} = h_n - \beta_n p_n, \quad p_n = J'u_n + \gamma_{n-1}p_{n-1},$$  \hspace{1cm} (13)$$

$$\beta_n = \frac{(J'h_n, p_n)}{\|A'_h p_n\|^2}, \quad \gamma_{n-1} = \frac{\|J'h_n\|^2}{\|J'h_{n-1}\|^2}, \quad p_0 = J'h_0$$  \hspace{1cm} (14)$$

The Frechet derivative of operator $A$ in the point $h \in H$ and operator conjugate to it are determined by the equations:

$$A'_h \Delta h = -h\Delta h^* + \cos \Theta h \Delta h^* - (h^* + \sin \Theta \left(1 + 2\overline{h}_0^3/h^3\right) - \cos \Theta h^*) \Delta h$$  \hspace{1cm} (15)$$

$$A'_h \xi = (h \xi)^* - \cos \Theta h \xi^* - (h^* + \sin \Theta \left(1 + 2\overline{h}_0^3/h^3\right) - \cos \Theta h^*) \xi$$  \hspace{1cm} (16)$$

Note that the Frechet derivative of the operator $A$ in the point $\overline{h}_0$ is:

$$A'_{\overline{h}_0} h_1 = -\overline{h}_0 h_1^* + \cos \Theta \overline{h}_0 h_1^* - 3 \sin \Theta h_1$$  \hspace{1cm} (17)$$

and the equation

$$-\overline{h}_0 h_1^* + \cos \Theta \overline{h}_0 h_1^* - 3 \sin \Theta h_1 = f$$  \hspace{1cm} (18)$$

has the meaning of linearized Eq. (8) for small film thickness changes. So $\overline{h}_0 + h_1(x)$ is the approximate solution of Eq. (8). The solution of this equation can be represented as convolution of $F(x)$ with $f$:

$$h_1(x) = \int_{-\infty}^{\infty} F(x - \tau) f(\tau) d\tau$$  \hspace{1cm} (19)$$

where $F(x)$ is fundamental solution of linear differential operator $A'_{\overline{h}_0}$.

The characteristic polynome of the operator $A'_{\overline{h}_0}$ is $-\overline{h}_0 \lambda^3 + \cos \Theta \overline{h}_0 \lambda - 3 \sin \Theta = 0$. The roots of this polynome are $\lambda_1 = -a, \lambda_{2,3} = a/2 \pm ib$, where $a, b$ are real positive numbers. $F(x)$ is composed by the Fourier transformation method and is expressed through the characteristic polynome roots as:

$$F(x) = \frac{4(\chi(x)e^{-ax} + \chi(-x)e^{-ax/2}(\cos(bx) - 3a/2b \sin(bx)))}{\overline{h}_0 (9a^2 + 4b^2)}$$  \hspace{1cm} (20)$$

where $\chi(x)$ is the Heaviside function.
For large film deformations (in comparison with $h_0$) linearized equation solution can substantially differ from the exact solution of Eq. (8). Let us find numerically the non-linear equation solution. Operators $A_i, A_i', A_i''$ are approximated by finite difference schemes of the first order of $\Delta h_x$:

$$A h_i = -h_i \left( -h_i - 2h_{i-1} + 2h_{i+1} - 2h_{i+2} \right) \frac{1}{2\Delta h_x} + h_i \cos \Theta (h_{i-1} - h_{i+1})/2\Delta h_x + \sin \Theta h_i$$

$$A_i p_i = -h_i \left( -p_{i-1} + 2p_{i-2} + 2p_{i+1} + p_{i+2} \right) \frac{1}{2\Delta h^3} + h_i \cos \Theta (p_{i+1} - p_{i-1})/2\Delta h_x - \left[ h_{i+2} + 2h_{i+1} - 2h_{i+1} + h_{i-2} \right] \frac{1}{2\Delta h_x^3}$$

$$- \sin \Theta \left( l + h_0^3 / h_i^3 \right) - \cos \Theta (h_{i+1} - h_{i-1})/2\Delta h_x f_i$$

where $h_i$ are the values of $h$ in the grid nodes, $\Delta h_x$ is the grid step, $z_i = A h_i - f_i$ is the residual vector, $p_i$ is descent direction of conjugate gradient method. The calculation is performed while $\|A h - f\|_{L_2}/\|f\|_{L_2} > \delta$, in our calculations $\delta = 10^{-6}$.

Figures 4 è 5 present film thickness and stream function calculated. The scales of axes are not equal, $h_0/\sigma = 0.065$. The numerical calculation shows the maximum of film thickness to be displaced up the flow from the point $x/\sigma = 0$ with maximal value of grad$T$. The difference between linearized equation solution and exact solution of Eq. (8), as it can be seen from Fig. 4, is considerable. The linearized equation solution is useful for numerical calculation when choosing calculation domain. The most increase of the film thickness in the region of the roller is 79% (at the maximum surface temperature gradient 13.6 K/mm). From the curves of stream-lines one can see that there is a reverse flow of liquid contrary the main flow in the roller. For all the experiments where the formation of regular structures was observed the calculations showed that there was a reverse flow of liquid in the horizontal liquid roller.

Fig. 4. Film thickness for the flow of 25% ethyl alcohol mixture ($\Theta = 90^\circ$, Re=2, $q=5.2$ W/cm$^2$, max $|T_x| = 13.6$ K/mm). 1 - numerical solution of Eq. (7), 2 - solution ignoring surface tension, 3 - solution of linearized equation (7).
CONCLUSIONS

1. By means of infrared thermography the information about temperature and temperature gradient distributions on the surface of the liquid film flowing down the plate with local heat source was obtained in wide range of regime parameters of experiment (film thickness, heat flux density, angle of plate inclination). It is showed that the regular structures on the falling liquid film surface under local heating have thermocapillary nature.

2. Numerical calculations of liquid motion within thin layer approximation are made for the temperature distribution measured in the experiment. It is established that in the horizontal liquid roller there is a thermocapillary reverse flow. Calculation predicts that the value of maximum deformation of liquid interface is 60-100% of initial film thickness.

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NOMENCLATURE

\( A \) operator

\( h \) film thickness, m

\( h_0 \) initial film thickness = \((3 \Gamma \nu / \rho g \sin \Theta)^{1/3}\), m

\( \bar{h} \) - film thickness, \( \bar{h} = h / l_\sigma \), dimensionless

\( g \) gravitational acceleration, m/s\(^2\)

\( l_\sigma \) capillary length \( \sqrt{\sigma / \rho g} \), m

\( L \) heater length, m

\( q \) heat flux density, W/m\(^2\)

Re film Reynolds number = \( G / \mu \), dimensionless

\( T \) temperature, °C

\( T \) dimensionless temperature

\( \dot{T} \) dimensionless temperature gradient

\( T_{\text{max}} \) - maximal film surface temperature, °C

\( u, v \) velocities along axes \( x, y \), m/s

\( u_0, v_0 \) characteristic velocities along axes \( x, y \), m/s

\( x, y, z \) Cartesian coordinates, m

\( X, Y, Z \) Cartesian coordinates, dimensionless

Greek symbols

\( \dot{\lambda} \) film flow rate per unit width, kg/m s

\( \Theta \) inclination angle, rad.

\( \Lambda \) distance between rivulets, m

\( \mu \) dynamic viscosity, kg/m s

\( \nu \) kinematic viscosity, m\(^2\)/s

\( \rho \) liquid density, kg/m\(^3\)

\( \sigma \) surface tension coefficient, N/m

\( \sigma_\tau \) surface tension temperature dependence coefficient N/mK

\( \psi \) - streamline function

Subscripts

\( 0 \) initial value

\( T, x, y, \tau \) derivative with respect to \( T, x, y \) and tangential.
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