CONJUGATED HEAT TRANSFER AT FLOW CONDENSATION IN MINICHLANNE WITH LONGITUDINAL FINS

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ABSTRACT

A mathematical model for vapor condensation in cylindrical longitudinally finned minichannel has been proposed taking into account wall temperature non-uniformity. The model describes annular and rivulet flow regimes until complete grooves flooding. It is shown that when the model does not account for the wall heat conductivity the maximal calculated enhancement of heat transfer is predicted for the “sharp” trapezoidal fins. The most enhanced heat transfer in the case of non-isothermal fins has been obtained for the curvilinear fins of expanded Adamek’s parametric family. When reducing the heat conductivity the curvilinear fins become still more effective in comparison with the trapezoidal fins. The wall non-isothermity is a factor, which cannot be neglected when modeling condensation in a minichannels with finned surfaces.

INTRODUCTION

The large-scale satellite and the International Space Station have made great progress in the past several decades. One of the problems urgent to be solved is the heat dissipation. There exists a large amount of heat that should be transferred and radiated into the outer space. Single-phase liquid loops were the major method used in large-scale spacecrafts for heat transfer and dissipation in the past decade. Since 1980s researches worldwide have focused their efforts on two-phase liquid loop technology to be used in the spacecraft thermal control systems on the International Space Station, telecommunication and technological satellites.

Loop Heat Pipe and Capillary Pumped Loop with “natural” circulation of two-phase flow are used on satellites to ensure the thermal transfer from core module equipments to a radiator. LHP and CPL are considered as reliable thermal management devices that are able of operating at any orientation in a gravitational field and heat can be transported over long distances. The main components of LHP and CPL are evaporator that is responsible for the generation of capillary forces that drive the working fluid via a porous structure and condenser (Gerasimov et al., 1975).

Most of the condensers for space applications have axial grooves and are made of extruded aluminum (Van Oost and Bekaert, 1996). The common working fluid is ammonia as its operational temperature is well suited to space applications (-40°C, +80°C). Enhancement of heat transfer at vapor condensation is of great importance for thermal stabilization systems in space applications. Keschock and Sadeghipour [1983], Da Riva and Sanz [1991] have proved that the length of smooth in-tube condenser under the microgravity conditions significantly exceeds that for similar condenser under ground conditions.

Enhancement techniques used on the outside of horizontal and vertical tubes in terrestrial conditions are generally applicable to in-tube vapor condensation. For a current review of the field see papers by Bergles [1999], Shah et al. [1999], Yang [1999] as well as the book by Webb [1994]. At low vapor velocities, the enhancement requirements for horizontal tubes are different, because gravity force drains the film transverse to the flow direction. Augmentation techniques employed include twisted-tape inserts, internal fins, and integral roughness. Internal fins have yielded the highest augmentation levels.

Considering stationary vapor condensation on individual fins and in the grooves is important branch of theoretical studies. Gregorig [1954] first derived a theoretical model for condensation on curvilinear fin with account of surface tension. The fin was described by equation for a spiral with a variable direction of rotation. Later models for stationary vapor film condensation on parametrical family of curvilinear fins with power dependent curvature have been developed by Bromley et al. [1966], Zener and Lavi [1974], Adamek [1981]. Markowitz et al. [1972] have proposed a theoretical model for condensation on inversed horizontal surface with two-
dimensional finning. Hirasawa et al. [1980] have developed a theoretical model describing hydrodynamics and heat transfer at film condensation on a vertical finned surface. The fin peak was of parabolic form with a small curvature radius. More detailed information concerning this problem one can find in the book by Webb [1994]. Theoretical research for dynamic and heat transfer aspects of condensate film flowing in narrow gaps was carried out by Nakoriakova et al. [1982].

Microfinning is being used widely last decade providing significant heat transfer enhancement, Webb [1999]. Spiral microfins, Bergles [1999], result in additional enhancement due to the flow swirling. Chamra et al. [1996] and Tang et al. [2000] have proved that heat transfer enhancement is higher in tubes with crossing grooves (herringbone) than that in tubes with spiral microfins. Ebisu [1999] has used the microfin herringbone tubes that give rise to irregular distribution of condensing liquid over the internal tube surface. Thin liquid film areas arise with enhanced vapor condensation (Cavallini et al. [2002]). Usually fin height equals to 100-200 μm. We suppose that higher fins are likely to be more effective in Microgravity as a result of small thin film thickness on the fins peaks and of condensate gathering in the grooves. For tubes with internal microfinning, in case of uniform condensate distribution over the tube perimeter, the fins would be flooded.

Nowadays there are no universal models published for condensation inside finned tubes with account of liquid capillary motion over the fins. We can just refer to Yang and Webb [1997] and Wang et al. [2002], who proposed the models for condensation inside tubes with axial and spiral finning.

The goal of this work is to propose a mathematical model for hydrodynamics and heat transfer in longitudinally finned condenser with account of wall nonisothermality. As condensation occurs in a tube, convective effects occur also, which are associated with the influence of shear stress on the liquid-vapor interface. The highest vapor shear effects arise near the inlet end into the tube, where vapor velocity is highest. We treat condensation under Microgravity conditions in the portions of annular and rivulet regimes of liquid flow inside tube. The basis of the presented model is founded on separate description for vapor flow dynamics and heat transfer. Solutions are obtained for laminar flow.

GEOMETRY OF THE FINNED TUBES

A peculiarity of condensation in longitudinally finned minichannels is a complicated effect of the channel’s form on the film dynamic because of capillary forces. Dynamic and heat transfer parameters of the process depend on the number, width and depth of grooves. In addition, main equations of mathematical models and opportunities for their facilitating can vary significantly for various tube geometry.

Fins of two types are considered: trapezoidal fins with round tips, Fig 1. a); and fins of Adamek’s parametrical family, Fig 1. b). Trapezoidal fins defining geometrical parameters are: \( R_{\text{ext}} \) - outside tube radius; \( R_f \) - the internal tube radius; \( R_m \) – the internal tube radius to the outside of the fins; \( a_f \) - the groove width; \( N \) - number of fins. Following conditions are met for the finning: 1. Lateral walls of the grooves are parallel; 2. Tip of the fin is an arc that mates with the lateral walls.

For Adamek’s parametrical family the derivative of the curvature, which defines the capillary pressure gradient, is defined as

\[
\kappa'(s) = a \left( s/S_i \right)^2 S_i^{-2}.
\]

Curvature has the form

\[
\kappa(s) = a \left( \xi + 1 \right)^{1-1/2} \left( s/S_i \right)^{1+1} + b/S_i .
\]

Angle between the vertical and radius of curvature (curve rotation) is

\[
\Theta(s) = a \left( \xi + 1 \right)^{-1} \left( s/S_i \right)^{1-1} + b/s/S_i .
\]

The curve-defining parameters are: \( \xi \) - power index (geometrical parameter describing the form of the fin), \( S_i \) - coordinate of inflection point, \( \omega \) - Angle between the vertical radius and curvature at the inflection point (maximal curve rotation). The conditions \( \kappa(S_i) = 0 \) and \( \Theta(S_i) = \omega \) mean that for \( \xi \neq -1 \) the parameters \( a \) and \( b \) may be expressed as

\[
a = -\omega \left( \xi + 2 \right) \quad b = \omega \left( \xi + 2 \right)/ \left( \xi + 1 \right) .
\]

If \( \xi = -1 \) then \( a = -\omega \), \( b = 0 \) and the fins became logarithmic one and the following formulas are valid:

\[
\kappa(s) = a \ln \left( s/S_i \right)/S_i , \quad \Theta(s) = a \left[ s \ln (s/S_i)/S_i - s/s_i \right] .
\]

Internal finning for this family is described in the following manner. Geometrical parameters that describe the fins are: \( R_{\text{ext}} \) \( R_f \) \( N \) \( a_f \) \( \xi \) \( K_r = S_m/S_i \). So the angles are calculated as

\[
\phi = \pi/N , \quad \psi = \phi - \arcsin \left( a_f /2R_f \right) .
\]

Equation

\[
O_f S_m = X_f (S_m) = R_f s \sin \psi
\]
is the main equation for fitting N fins, where coordinate \(X_f(s) = \int_0^s \cos(\Theta(s)) \, ds\). From this equation the total length of the fin \(S_n\) is derived.

The fin height, the effective width of the groove, the internal tube radius \(R_n\) and the groove depth are calculated using formulas:

\[
FO = Y_f(S_n) \text{, } a_z = \mathcal{S}_z = a_f + 2(X_f(S_n) - X_f(S)) \text{, }
\]

\[
[FO] = [OE] - [FO] - [OE] = R_f - Y_f(S_n) - R_f(1 - \cos \psi)
\]

\[
h = S_n H = [OF] \cos \phi + ([OS] - [OF] \tan \phi) \sin \phi.
\]

Where coordinate \(Y_f(S) = \int_0^s \sin(\Theta(s)) \, ds\).

It is assumed that:

1. The total length of the fin \(S_n\) is obtained in a way to fit the fins into the tube according to their number \(N\) and the groove width \(a_f\). This requirement is expressed by the equation \([S,S] = R_f \sin \psi\).
2. The angle between the vertical and radius of curvature at the inflection point is equal \(\omega = \theta(S_n) = \pi/2 - \phi\), it means that the tangent lines at the inflection points for neighbor fins are parallel.

### VAPOR-LIQUID FLOW IN THE CHANNEL

**The problem statement and basic equations.** A schematic model of the process is shown in Fig. 2. The vapor flow area is divided in two parts: 1v – region of the axially symmetric vapor flow in the central part of the channel; 2v – vapor flow in a groove. The condensate flow area is divided into three parts: 1L – thin liquid film on the curved surface of the peak of the fin; 2L – thin liquid film on lateral surface of the fin; 3L – condensate in the groove (flooding height varies along the channel).

Generally, vapor velocity in the central part of the channel may depend on azimuth angle. It is assumed that for fins with sharp peaks a contact area between vapor flow core (1v) and liquid film at upper part of the fin (1L) is small. To simplify the basic equations it is assumed that:

1. Vapor flow is laminar through the whole length of the channel.
2. The Stocks equation is valid for the vapor flow in the groove.
3. Condensate flow is viscous that permits to neglect the inertial terms in the Navier-Stokes equation.
4. The vapor flow in the central part of channel contacts only with vapor moving in grooves so the vapor shear stress at the bound \(r = R_n\) is equal to vapor shear stress in the 2v region.
5. Capillary pressure exceeds vapor shear stress in the 2L-region. Liquid velocity in the \(y\)-direction is substantially higher than that in the \(z\)-direction.
6. The depth of the groove \(h_f\) significantly exceeds the width of the groove, which permits to apply the narrow-channel approximations for liquid flow in the groove.

It is assumed that the width of the peaks is equal to zero and its surface added to an effective height of the fin \(\tilde{h}_f\). The grooves are set to be rectangular. New definitions for the height of the fin \((\tilde{h}_f)\) is equal to one-half of the heat-transfer surface of the fin) and for the internal tube radius \((\tilde{R}_n = R_f - \tilde{h}_f)\) are used. In the 1v-region we have

\[
ρ_x \left( u_x \frac{\partial u_x}{\partial z} + u_x \frac{\partial u_x}{\partial r} \right)_w = -\frac{dP}{dz} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu_x \frac{\partial u_x}{\partial r} \right)_w + \rho \left( u_x \frac{dH}{dz} - u_y \right)_w.
\]

In the 2v and 3L regions continuity Eq. (7) is used with equation

\[
\frac{\partial p}{\partial z} = \mu \Delta u,
\]

Boundary conditions for 3L-region have the following form: non-slip condition at the wall \((y = 0, x = 0)\) \(u_x = u_y = u_z = 0\); conjugated conditions at the film-vapor boundary \((y = H)\)

\[
u_{x\mid y=H} = u_y, \quad \mu_x \frac{\partial u_x}{\partial y} = \mu_x \frac{\partial u_x}{\partial y} = \tau_s,
\]

\[
\rho \left( u_x \frac{dH}{dz} - u_y \right)_w = \rho \left( u_x \frac{dH}{dz} - u_y \right)_w.
\]

Boundary conditions for 2v-region have the following form: non-slip condition at the wall \((x = 0)\) \((u_y)_x = 0\). Conjugated conditions between 1v- and 2v-regions at \(r = \tilde{R}_n\):

\[
(u_x)_y = (u_x)_y, - (\mu_x \frac{\partial u_x}{\partial y})_y = (\mu_x \frac{\partial u_x}{\partial y})_y.
\]

This system of equations is solved analytically. It is neglected by the left part of Eq. (6) on this stage of calculations. It assumed that condensation does not effect on vapor flow dynamics. In 2v and 3L-regions liquid is considered to flow in two directions: in \(z\)-direction under pressure gradient \(dp/dz\) and shear stress at the boundary between 1v- and 2v-regions and between 2v- and 3L-regions, as well as in \(y\)-direction. Flow rate in the \(y\)-direction at upper part of the groove is equal to the rate of condensed vapor in the channel. Vapor flow rate is reducing continuously due to condensation at the walls of the fins.
Solution for the equations (7)-(8) in the groove, far away from the central part of the channel has the form

\[ u_z = u_{z0} \left[ 1 - \left(2x/a_f \right)^2 \right], \]

where \( u_{z0} \) is an unknown function dependent on \( y \) only.

Considering vapor motion in the groove (2v-region) downward \( z \)-direction it is assumed that \( u_z \) does not depend on \( z \). It follows from (8) that

\[ \frac{dp}{dz} = \mu_t \left( \frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} \right). \]

Substituting (11) to (12) gives after integrating over \( x \) from \(-a_f/2\) to \( a_f/2\)

\[ \frac{dp}{dz} = \frac{8\mu_t u_{z0}}{a_f^2} - \mu_t \frac{2}{3} \frac{d^4 u_{z0}}{dy^4}. \]

For the 3L-region similarly we get

\[ \frac{dp}{dz} = \frac{8\mu_t u_{z0}}{a_f^2} - \mu_t \frac{2}{3} \frac{d^4 u_{z0}}{dy^4}. \]

Equations (13) and (14) transforms like in Nakoriakov et al. [1982] as

\[ \frac{d^2 \Phi}{dy^2} + \frac{12\Phi}{a_f^2} = 0, \]

where \( \Phi = u_z + a_f^2/8\mu_t (dp/dz) \) for vapor and \( \Phi = u_{z0} + a_f^2/8\mu_t (dp/dz) \) for condensate. Solution of the Eq. (15) can be written in the form

\[ \Phi = C_1 \exp(-k_1y) + C_2 \exp(k_2y). \]

We can obtain an approximated solution of the Eqs. (6)-(10).

For more precise evaluation of the pressure the equations (6)-(7) for vapor flow in the central part of channel are deducted taking into account the convective terms in the left part of Eq. (6) as proposed in Mihalevich [1982]. For the parabolic velocity profile it is obtained following equation:

\[ \frac{dp}{dz} = \frac{2x}{R_m} + \frac{2.66\overline{\mu}_t - u_z(R_m)}{\pi R_m^2} \frac{dG_{ci}}{dz}, \]

which is used for pressure evaluation.

**HEAT TRANSFER AND CONDENSATION**

**Annular flow.** It is assumed that there is two heat transfer modes at condensation inside a finned tube. For the first mode corresponding to the annular flow at the initial part of the channel a smooth film of condensate arise all over the wetted perimeter of a channel. Gravity and capillary forces effects can be neglected. Heat transfer in this case is similar to that for condensation of moving vapor on a plane plate. As a characteristic velocity of vapor the average vapor velocity in the grooves is taken. A similar problem has been solved by Cess [1954]. A balance between vapor-liquid shear stress and capillary forces defines a length of the annular flow. This may be expressed by the threshold film thickness value.

**Rivulet flow.** For the second mode it is assumed that in a thin film of condensate on the fin, capillary forces are much higher than vapor-liquid shear stress, and the fin surface is non-isothermal. Transition from the annular flow mode to the rivulet flow mode is specified assuming that the effective liquid film thickness at the end of the first one equals to that at the beginning of the second one.

The calculations are performed step by step for sections \( \Delta x \) along the channel. As a result of the dynamic problem solution we get a function of condensate flow rate \( G_i(H) \) of groove flooding \( H \) for any fixed total mass flow rate \( G \). The condensate flow rate along coordinate \( z \) can be expressed as follows

\[ G_i(z) = G_{ic} + \int_{z_0}^z dG_i(H(\zeta)). \]

where \( dG_i(H(\zeta)) \) is the amount of liquid condensed in the channel section \( \zeta + d\zeta \) under the groove-flooding level \( H(\zeta) \).

\( G_{ic} \) is the flow rate of the liquid condensed in the initial part of the channel \([0,z_0]\) calculated according to the model for annular flow. So the equation for \( H(z) \) is obtained as

\[ G_i(H(z)) = G_{ic} + \int_{z_0}^z dG_i(H(\zeta)). \]

**Vapor condensation on a non-isothermal fin.** It is considered that \( R_m/R_f << L \), i.e. the maximal wall thickness is much less than the channel length, then the heat fluxes lengthwise the channel could be ignored. In this case the set of two-dimensional problems could be solved for each specific section \( \Delta x \). Besides, if the outside channel wall temperature is constant then in view of periodicity and symmetry we may consider just a half of a fin. This facilitates extremely the calculations.

The steady-state flow of condensed liquid film is defined by capillary pressure gradient and viscous shear stress. It is described in lubrication approximation by the following equation

\[ \mu \frac{\partial^2 u}{\partial y^2} = \frac{dp}{ds}. \]

A steady-state temperature distribution within the fin and the film is defined by the Laplace’s equation

\[ \Delta T = 0. \]

Mass flow rate through the film surface equals to the condensate flow rate:

\[ \int_0^1 \frac{\lambda T_y(s,\delta(s))}{R_f} ds = \int_0^1 \rho u(s,\delta(s)) ds. \]

This equation has been obtained as a result of combining the continuity equation with boundary conditions at the film surface. Boundary conditions are as follows:

\[ u(s,0) = u_z(s,\delta(s)) = 0, T_{i0} = T_{i00} = 0, T_{i0x} = T_{i0}, T(s,\delta(s)) = T_f. \]
As the film is thin the streamwise conductivity can be neglected in the Eq.(20). Integrating (19)-(20) with account of the boundary conditions (22) gives expressions for velocity and temperature profiles in the film for the known wall temperature \( T_w(s) \):

\[
u(u, y) = -\frac{1}{\mu} \frac{d}{ds} \left( \frac{\lambda}{y} \left( \delta(s) - \frac{y}{2} \right) \right), T(s, y) = T_w(s) + \frac{\Delta T(s)}{\delta(s)} y, \quad (23)
\]

where \( \Delta T(s) = T_r - T_w(s) \). Inserting expressions (23) into Eq. (21) yields the equation for the condensate flow rate along the film surface

\[
m(s) = \frac{\lambda}{r_w \rho} \int_0^l \Delta T(s) \delta(s)^{1/3} ds = -\frac{\rho \delta(s)^{1/3} dp}{3 \mu} ds. \quad (24)
\]

Integrating Eq. (24) leads to relations for film thickness and condensate flow rate

\[
\delta(s) = \left( \frac{dp}{ds} \right)^{-1/4} \left[ \int_0^s \frac{\lambda \Delta T(t)}{r_w \rho} \left( \frac{dp}{ds} \right)^{1/3} dt \right]^{3/4}, \quad (25)
\]

\[
m(s) = \frac{\rho}{3 \mu} \delta(s)^{1/4} \left[ \int_0^s \frac{\lambda \Delta T(t)}{r_w \rho} \left( \frac{dp}{ds} \right)^{1/3} dt \right]^{3/4}. \quad (26)
\]

**Thermal conductivity in a fin.** Temperature distribution in a fin obeys the Laplace equation (20) with the following boundary conditions. On the external tube surface temperature \( T_\infty \) is specified

\[
T\big|_{s=0} = T_\infty. \quad (27)
\]

Due to fins periodicity we get

\[
\frac{\partial T}{\partial \theta} \big|_{\theta=0} = \frac{\partial T}{\partial \theta} \big|_{\theta=\pi} = 0. \quad (28)
\]

Let \( q(s) \) to be a heat flux on the fin surface where condensation takes place

\[
\lambda_w \frac{\partial T}{\partial n} \big|_{X>0, y=X/T(s)} = q(s). \quad (29)
\]

The heat flux \( q(s) \) should be obtained from the condensation problem (19)-(22). From (23) we get

\[
q(s) = \frac{\lambda}{r_w} \frac{\Delta T(s)}{\delta(s)}. \quad (30)
\]

Thus

\[
\lambda_w \frac{\partial T}{\partial n} = \frac{\lambda}{r_w} \frac{\Delta T(s)}{\delta(s)}. \quad (31)
\]

And taking into account (25)

\[
\lambda_w \frac{\partial T}{\partial n} = \frac{\lambda}{r_w} \frac{\Delta T(s)}{\delta(s)} \left( \frac{dp}{ds} \right)^{1/4} \left[ \int_0^s \frac{\lambda \Delta T(t)}{r_w \rho} \left( \frac{dp}{ds} \right)^{1/3} dt \right]^{3/4}. \quad (32)
\]

A boundary value problem (20), (27), (28) with nonlinear boundary condition (32), which describes the film condensation on curvilinear film surface, is obtained.

There are two unknown parameters \( dp/ds \) and \( \delta(s) \), which depends on fin shape. For curvilinear fins from Adamek’s family \( dp/ds \) is defined by following equation

\[
dp / ds = \sigma \kappa \left( s / s_1 \right)^2 S_0^2. \quad (33)
\]

For trapezoidal fins with round tips a model proposed by Markowitz et al.,1972 is used. So the following equation defines the film thickness on the round fin tips

\[
\delta \left[ \frac{d^2 \delta}{d \Psi^2} \right] = \frac{12 \mu R_1^4 k \Delta T(\Psi)}{r_w \rho \sigma}. \quad (34)
\]

Boundary conditions at \( \Psi = 0 \) are the same as in Markowitz et al.,1972:

\[
\delta \big|_{\Psi=0} = \frac{3 \mu R_1^4 \Delta T(\Psi)}{r_w \rho \sigma}, \quad \frac{\partial \delta}{\partial \Psi} \big|_{\Psi=0} = 0. \quad (35)
\]

For lateral side of the trapezoidal fin the following equation is used

\[
\frac{\delta}{\partial s} \left[ \frac{dp}{ds} \right] = \left( \frac{\sigma}{R_1 + \sigma} \right)^\frac{1}{\psi} \left[ \frac{\delta}{\partial \Psi} \right] \left[ \psi - H \right], \quad (36)
\]

where \( R_1 \) is a curvature radius of the film surface at the end of the round fin tip and \( R_2 = a_j / 2 \) is a half of the groove width.

**NUMERICAL RESULTS AND DISCUSSION**

Average and local heat transfer rate, pressure drop, vapor and condensate velocities, vapor void fraction and other parameters have been calculated numerically using the proposed model for steam condensation in longitudinally fined tubes. All tubes have 16 fins. The groove width and the fin height for trapezoidal fins with round tips are equal 1 and 3 mm respectively (see rows 8-10 of Table 1). For the vapor flow rate \( G = 0.0113 \text{ kg/s} \) the pressure drop rises 5.2 times (row 2 in Table 1). The tube section concerned lengthened from 1.1 m to 2.7 m, i.e. 2.4 times. For the vapor flow rate \( G = 0.0113 \text{ kg/s} \) the pressure drop comes to 0.33 bar that causes significant change of saturation temperature along the tube equal to 3K.

The model predicts a short initial section of annular flow mode and relatively long section of rivulet flow mode. The transition is marked in the Fig. 6b by the break of the curves for average heat transfer coefficient versus \( \psi \)-coordinate. Vapor flow rate increasing brings to extension of rivulet flow region and pressure drop rise. Average heat transfer coefficient reduces slightly. When vapor flow rate increases twice the pressure drop rises 5.2 times (row 2 in Table 1). The tube section concerned lengthened from 1.1 m to 2.7 m, i.e. 2.4 times. For the vapor flow rate \( G = 0.0113 \text{ kg/s} \) the pressure drop comes to 0.33 bar that causes significant change of saturation temperature along the tube equal to 3K.

Reducing of initial pressure leads to the average heat transfer coefficient increasing (row 3 in Table 1). It is caused by vapor velocity rise at the tube entry for \( G = \text{const} \).
vapor density reducing. There are 98 percents of vapor being condensed at the annular and rivulet flow modes that have a short length equal to 0.78 m. The pressure drop comes to 0.17 bar. A reducing of the initial temperature difference lengthens significantly the annular and rivulet flow part of the tube (up to 5.5 m), while the average heat transfer coefficient increasing (row 4 in Table 1).

Fig. 5 shows temperature distribution in fins made from different materials. Dimensionless temperature is changing from 0 that corresponds to the outer side of the tube, to 0.99 on the top of the fins made of stainless steel. Essential effect of the wall thermal conductivity on condensation in comparison with isothermal fins is observed (rows 1, 5-7 in Table 1). With decreasing of the thermal conductivity coefficient the tube length to the grooves flooding increases, and the average heat transfer coefficient calculated on the base of initial temperature difference reduces. So for aluminum the tube length to the complete grooves flooding increases to 1.75 m and the average heat transfer coefficient reduces from 18000 to 11000 W/m²K. For stainless steel the tube length increases to 32 m and the heat transfer coefficient reduces by a factor 23.

The numerical results for curvilinear fins are presented in Table 1 (rows 8-10) and in Fig. 6. Mass vapor content comes to 0.0-0.02 by the moment of the grooves flooding, i.e. almost complete condensation of vapor takes place. The length of the tube comes to 0.79 m in the case of isothermal curvilinear fins that is by a factor 1.4 less than one in the case of trapezoid fins with rounded tips, and by a factor 1.5 longer in comparison with the “sharp fin”. The remarkable enhancement of vapor condensation in comparison with trapezoid fins takes place and the average heat transfer coefficient comes to 26 000 W/m²K in the case of isothermal curvilinear fins.

When the wall heat conductivity is taken into account the most enhanced heat transfer is predicted for the curvilinear fins. For tubes made of copper the curvilinear fins provide the same heat transfer intensity and tube length as “sharp” trapezoidal fins with height equal to 4 mm (compare rows 5, 9 and 12). But the pressure difference is smaller for curvilinear fins. The curvilinear fins are most effective when the heat conductivity reduces ($\lambda_w=150$ W/mK). The tube length to the point of the grooves flooding in this case comes to 1.39 m and by a factor 2 less than one in the case of “sharp” trapezoid fins.

### Table 1. Parameters of calculated regimes.

<table>
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<th>#</th>
<th>$G_w$, kg/s</th>
<th>$P$, MPa</th>
<th>$\Delta T$, K</th>
<th>$\lambda_w$, W/mK</th>
<th>$Z_1$, m</th>
<th>$\alpha$, W/m²K</th>
<th>$\Delta P$, kPa</th>
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$\infty^*$ Infinite heat conductivity corresponds to isothermal internal tube surface.

Calculations have shown that in the case of condensation inside tubes with relatively high longitudinal fins average vapor
velocity in the central part differ from that in the interfin channels by a factor 10 (see Fig. 6 c and d). The superficial vapor velocity used for engineering estimations may not represent well the interaction between vapor and film of condensate. The velocity of condensate in the channel first increases and than reduces in all cases while the vapor velocity in the central part of the tube and in the interfin channels is decreasing monotonically.

It is shown that when the model does not account for the wall heat conductivity the maximal calculated enhancement of heat transfer is predicted for the “sharp” trapezoidal fins. In the case of non-isothermal fins it is shown that the most enhanced heat transfer is achieved for the curvilinear fins. For the “sharp” fins the values of calculated average heat transfer coefficients in the case of isothermal wall differ by a factor 10 from those obtained with account of the wall heat conductivity. The general conclusion is that the wall non-isothermality is a factor, which cannot be neglected when modeling condensation in a minichannel with finned surfaces.

CONCLUSIONS

1. A mathematical model for vapor condensation in longitudinally finned tubes has been proposed. The model considers condensation at two flow sections – annular and rivulet. It allows calculating the pressure drop, vapor and liquid velocity profiles, temperature in the tube wall and local and average heat transfer coefficients.

2. The calculations using the proposed model have shown that at vapor condensation in longitudinally finned tubes of relatively small diameter it is important to divide the cross-section of the channel into central part and the interfin channels at physical and mathematical modeling. The average velocities in these channels can differ by a factor 10.

3. For tubes with relatively high fins \( h > a_F \) the proposed model describes the almost complete condensation. Mass vapor content by the moment of the grooves flooding comes up to 0.01-0.05.

4. The most enhanced heat transfer in the case of non-isothermal fins has been obtained for the curvilinear fins of expanded Adamek’s parametric family. When reducing the heat conductivity the curvilinear fins become still more effective in comparison with the “sharp” trapezoidal fins.

Acknowledgements

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NOMENCLATURE

- \( a_f \) – distance between fins, m
- \( b_t \) – fin thickness at the peak, m
- \( b_f \) – fin thickness at the base, m
- \( c \) – liquid specific heat, kJ/kg/K
- \( G \) – mass flow rate, kg/s
- \( h \) – local heat transfer coefficient, W/m²/K
- \( h_f \) – fin height, m
- \( L_c \) – length of the channel, m
- \( l_c = \left[ \frac{\sigma (\rho - \rho_v) g}{\rho_v^2 \nu g} \right]^{1/2} \), capillary constant, m
- \( l_v = \left( \frac{\sqrt{g}}{\nu} \right)^{1/3} \) - scale of viscous-gravitational interaction, m
- \( N \) – number of fins,
- \( P, p \) – pressure, Pa
- \( P_w \) – wetted perimeter, m
- \( q \) – heat flux, W/sm²,
Greek symbols

- \( \delta \) - film thickness, m
- \( \lambda \) - thermal conductivity, W/m/K
- \( \mu \) - liquid dynamic viscosity, Kg/m/s
- \( \nu \) - liquid kinematic viscosity, m²/s
- \( \sigma \) - surface tension coefficient, N/m
- \( \rho \) - density of the liquid, kg/m³
- \( \Theta \) - Angle between the vertical and radius of curvature
- \( \phi \) - Angle between the vertical and radius of curvature at the inflection point

Subscripts

- \( \theta \) - indication of the initial flow parameters
- \( c \) - central part of the channel
- \( h \) - grooves
- \( v \) - vapor
- \( l \) - liquid
- \( w \) - channel wall

REFERENCES


